

# Contribution of Steel Shear Reinforcement in Two-Way Shear Resistance of Flat Plates (Parametric Study)

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**Abstract** – The current research presents an experimental, analytical and finite element investigation to study shear enhancement of flat plates by shear heads. Experimental results of nine half-scale concrete flat plate specimens were tested. Three different column aspect ratio were tested in the current study as follows: 1:1, 2:1 and 4:1. Two different lengths of shear heads from column face equal to  $2.00d$  and  $2.60d$  were examined with change of column aspect ratios in which " $d$ " is flat plate depth. Structural behaviors were evaluated in terms of overall load-deflection response, ultimate loading capacity and ductility. Failure patterns and strain distribution were also discussed. The research also presents parametric study through numerical analysis taken in to account most important variables which haven't been studied in the practical program. Different arrangement of shear head legs, changing cubic compressive strength of concrete were some of these variables. Several different lengths of shear heads were also studied. Experimental and numerical results were analyzed through regression analysis to propose a new formula to predict the ultimate punching shear capacity of flat plates with shear heads.

**Keywords** – Flat plates; Shear heads; Column aspect ratio; Finite element investigation; Parametric study; Numerical analysis; Regression analysis

## INTRODUCTION

Punching shear is one of the most important criteria that control design of flat plates. There are many traditional ways to enhance punching shear resistance of flat plates such that using thick slab and large column or drop panel utilization with and without column heads. The ACI 318 and CSA (Canadian Standards) codes are based mainly on Moe's work (1961), while the BS 8110 (British Standard Institution) and EC 2 codes are based on Regan's work (1974). The present building code specifications for the shear strength of reinforced concrete slabs are based on the test results of plates made with relatively low compressive strengths, varying mostly from 14 to 40 MPa. The Kinnunen and Nylander concrete punching rational model (1960) still provides one of the best accounts of the punching behavior of concrete plates without shear reinforcement. Modern European codes of practice treat punching shear in terms of shear stresses calculated at control perimeters located at relatively large distance from the column or loaded area. In the CEB-FIP 1990 model code, at a distance is  $2d$ . In BS 8110-97, it is  $1.5d$ , but the peripheral has square corners as compared with CEB-FIP rounded corners.

In North American codes such as ACI 318 and CSA Canadian the control punching shear peripheral is only  $0.5d$  away from the loaded column. Different European and North American codes and design guidelines allow the use of shear reinforcement for two-way slab plates. More recently, the American and the Canadian codes allows the use of shear studs developed by Ami Ghali and walter Dilger at Calgary University (1981). Also Punching shear reinforcement has become one of the approved methods in most codes of the Middle East countries such as Egyptian code of practice ECP 203(2017). Corley and Hawkins (1968) suggest details of a shear heads. This mechanism makes use of structural steel sections welded collectively to make a grid which can then be placed around or via a column. Their study shaped the basis of the shear head reinforcement design guidelines and recommendations in the American Code Institute design code ACI 318. The current research develops a new formula to predict contribution of shear heads in resisting punching shear of flat plates.

## CODE REQUIREMENTS FOR TWO-WAY SHEAR

Two-way shear design of flat slabs uses the critical sections (control perimeter) approach in most design code requirements. The nominal shear strength is determined at acritical section around the column or loaded area. Concrete strength and geometric parameters are usually governing the function of shear strength.

### ACI 318-14 CODE PROVISIONS

ACI 318-14 requires the factored shear stress  $v_c$  at the critical section (the perimeter at a distance  $\frac{d}{2}$  from column faces).

The factored shear resistance of the critical section is the smallest of the values obtained from eqs. [1] up to [3]:

$$v_c = 0.33 \phi \sqrt{f'_c} \quad (1)$$

$$v_c = 0.083 \phi \sqrt{f'_c} \left( 2 + \frac{4}{\beta_c} \right) \quad (2)$$

$$v_c = 0.083 \phi \sqrt{f'_c} \left( 2 + \frac{\alpha_s d}{b_o} \right) \quad (3)$$

Where  $\beta_c$  is the ratio of the long side over short side of the column,  $b_o$  is the perimeter length of the critical section.  $\alpha_s = 40, 30$  and  $20$  for interior, edge, and corner column, respectively.

### EURO CODE 2 PROVISIONS

The Euro code 2 (2004) displays a basic simple control section at a distance  $2d$  from the faces of the column or the loaded area. Further, the shear stress  $v_f$  on the control section should be no more than the shear resistance ( $V_r$ ).

$$V_f \leq V_r \quad (4)$$

As shown in Figure (1), for rectangular columns, the basic control section includes round corners (ACI and CSA code permit right angle corners). The code also requires checks on the column face and on the control section outside the shear reinforcement area. For interior slab-column connections without shear reinforcements, the shear resistance  $V_r$  for the basic control section is calculated as:

$$V_c = \frac{0.18}{\gamma_c} k (100 \rho_l f_{ck})^{1/3} 0.10 \sigma_{cp} \geq (V_{min} + 0.10 \sigma_{cp}) \quad (5)$$

$$k = 1 + \left( \frac{200}{d} \right)^{0.5} < 2, d \text{ in mm} \quad (6)$$

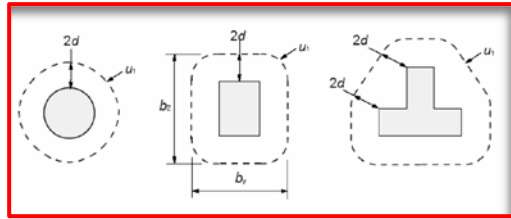


Figure 1: Critical Sections Defined in Euro Code 2 (2004)

$f_{ck}$  = the characteristic concrete strength, MPa

$$\rho_l = (\rho_z \rho_y)^{1/2} \leq 0.02 \quad (7)$$

$\rho_z, \rho_y$  are reinforcing ratios in z, y directions for a slab width equal to column width plus  $3d$  each side.  $\gamma_c = 1.5$ , partial factor for persistent and transient concrete.

$$V_{min} = 0.035 k^{3/2} f_{ck}^{1/2} \quad (8)$$

The shear stress  $v_f$  at the basic control section due to factored external concentric load  $V_f$ .

$$v_f = \frac{V_f}{u_1 d} \quad (9)$$

For interior columns, the shear stress " $v_f$ " at the column face is

$$v_f = \frac{V_f}{u_0 d} \quad (10)$$

Where  $u_0$  is the length of column perimeter (for interior column)

### ECP 203 CODE PROVISIONS

The ECP 203 (2017) employs a basic control critical section at a distance ( $d/2$ ) from the faces of the column.

$v_c$  = punching shear strength of concrete according to () ECP 203-2017

$v_c$  = The least value from the following equations:

$$v_c = 0.316 \left( 0.50 + \frac{a}{b} \right) \sqrt{\frac{f_{cu}}{\gamma_c}} \quad (11)$$

$$v_c = 0.316 \sqrt{\frac{f_{cu}}{\gamma_c}} \quad (12)$$

$$v_c = 0.80 \left( \frac{\alpha \cdot d}{b_o} + 0.20 \right) \sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{but not more than } v_c = 1.7 \text{ N/mm}^2 \quad (13)$$

Where ( $a/b$ ) is the ratio of the short side over long side of the column,  $b_o$  is the perimeter length of the critical section,  $\alpha = 4, 3, 2$  for interior, edge, and corner column, respectively.  $\gamma_c$  is reduction factor for compressive strength of concrete.

### PUNCHING SHEAR DESIGN OF FLAT SLABS WITH SHEAR HEADS REINFORCEMENT

#### ACI 318-14 CODE PROVISIONS

ACI 318-14 is the only code which predict the resisting punching force of flat slabs with shear heads, to increase shear strength by steel (I) shapes (shear heads), the following steps should be followed:

a- Calculate total factored shear force [ $V_u$ ] according to the following equation:

$$V_u = W_{su} \cdot A \quad (14)$$

where  $A = (L \cdot B) - (a \cdot b)$

b- Calculate shear strength ( $v_c$ ) without shear reinforcement in which ( $v_c$ ) is the smallest of the values obtained eqs. [1] up to [3].

c- Check the safety by comparing ( $V_u$ ) with ( $\Phi v_c$ ) where  $\Phi=0.75$

If the safety is not satisfied, it should be increase the shear strength by steel (I) shapes "Shear heads"

d- Check maximum shear strength permitted with steel shapes

$$V_n = \Phi (0.58 \sqrt{f_c'} b_{o1} d) \quad (15)$$

e- Determine minimum required perimeter ( $b_{o2}$ ) of a critical section at shear head ends with shear strength limited to

$$V_n = \Phi (0.33 \sqrt{f_c'} b_{o2} d) \quad (16)$$

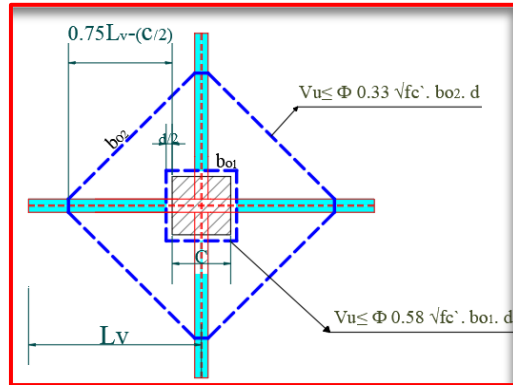


Figure (2): Allowable Punching Shear Strength at Different Perimeters  $b_{o1}$  and  $b_{o2}$ , f- Determine required length of shear head arm " $L_v$ " to satisfy  $b_{o2}$ .

Figure (2) shows allowable punching shear strength at different perimeters  $b_{o1}$  and  $b_{o2}$ . Strength of slab is reached; the plastic moment required to ensure that the ultimate shear is attained as the moment strength of the shear head is reached should be calculated as follows:

$$\Phi M_p = \frac{V_u}{2n} \left[ h_v + \alpha_v \left( L_v - \frac{c}{2} \right) \right] \quad (17)$$

Where:

$M_p$ : plastic moment strength for each shear head arm.

$\Phi$ : strength reduction factor for tension controlled member, equal to 0.9.

n: number of shear head arms.

$L_v$ : minimum required length of shear head arm.

$h_v$ : depth of shear head cross-section.

$\alpha_v$ : the ratio between the flexural stiffness of each shear head arm and that of the surrounding composite cracked slab section of width (c+d) [Assume  $\alpha_v=0.25$ ]

f- Determine required length of shear head arm " $L_v$ " to satisfy  $b_{02}$

g- Check depth limitation of shear head

web ( $h_v \leq 70 t_w$ ).

h- Determine location of composite flange of steel shape with respect to compression surface of slab in which compression flange must be located at  $0.30d$ .

## RESEARCH SIGNIFICANCE

The two-way slab system is a significant efficient structural system. It is economical and is widely used in different structural applications such as floors, roofs of buildings and walls of tanks. Shear heads is one of the most important Punching shear reinforcement. Contribution of shear heads in resisting punching shear of flat slabs defined only in ACI 318-14. In an attempt to propose a new design formula to predict punching shear capacity of flat slabs with shear heads. Experimental, numerical and analytical analysis were done taking column aspect ratio into consideration.

## EXPERIMENTAL PROGRAM

### MATERIALS

The concrete mix was provided by housing and building research center "HBRC" lab.

$$b_{02} = 4\sqrt{2}\left[\frac{c}{2} + \frac{3}{4}\left(L_v - \frac{c}{2}\right)\right] \quad (18)$$

To ensure that premature flexural failure of shear heads doesn't occur before shear

The concrete was designed to have a 28-days cube compressive strength of 25MPa.

Table (1) presents the mix proportions to produce one cubic meter of the concrete mix.

The tensile mechanical properties of the steel bars were tested according to ASTM A370-97a.

Table (2) presents the yield strength, ultimate strength and elongation.

Table (3) presents the yield strength, ultimate strength and elongation of steel in which shear heads made from.

### TEST SLABS

A total of nine half-scale flat slab specimens were tested, all tested flat slabs with overall thickness equal to 150 mm and span equal 2000 mm in both directions with clear spans between supported beams equal to 1800 x 1800 mm. A total of nine slabs with square and rectangle column heads, were tested under punching shear loading. The concrete cover used was 10 mm to the bottom face of all test specimens. All slabs were reinforced with bottom longitudinal steel bars mesh  $\Phi 18@100\text{mm}$  and top mesh with  $\Phi 10@200\text{mm}$ . All columns were reinforced with four longitudinal steel bars  $\Phi 18$  and confined with  $\Phi 10@100\text{mm}$  bars as transverse reinforcements. Table (4) summarizes the general description of the test specimens. Figure (3) shows specimens with steel shear heads during casting. Test setup of all specimens could be showed in Figure (4).

### ELECTRICAL STRAIN GAUGES

Four strain gauges were used to monitor the steel reinforcement strain in the two directions at column face as shown in Figure (5). Deflection of specimens are measured using five linear variable differential transducers, at mid-span and two at distance equal  $L/8=225\text{-mm}$  and  $L/4= 450\text{-mm}$  from the center in each direction as shown in Figure (6) where L is the distance between supports.

Also, Electrical strain gauges with 10 mm length,  $119.8 \pm 0.2$  ohms' resistance were used to measure shear strain in web and longitudinal strains of top and bottom flanges of steel shear head sections at  $d/2$  and  $0.75(L_v - c/2)$  from column face in an attempt for measure contribution of shear heads in punching shear and also to know however steel flanges reached to yielding or not, Figure (7) and (8) shows typical arrangement and configuration of strain gauges in web and flanges of steel shear sections, then at every stage of loading, radial and tangential cracks were observed and marked.



Figure (3): Specimens with Steel Shear Heads during Casting



Figure (4): Test Setup of Specimens.



Figure (5): Locations of Steel Strain Gauges.



Figure (6): Arrangement of LVDTs over the Specimen.



Figure (7): Electrical Strain Gauges of Steel Shear Heads.

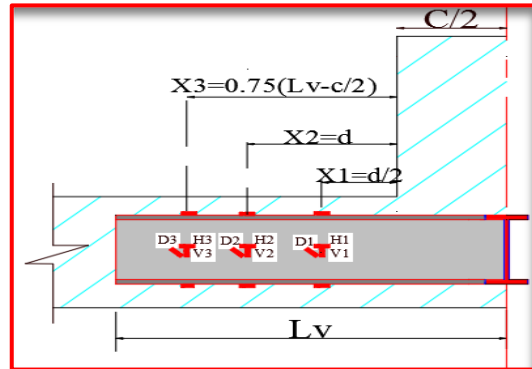


Figure (8): Typical Configuration of Strain Gauges in Web of Shear Heads.

## I. TEST RESULTS AND DISCUSSION

This section explains the test results and observed behavior of the tested flat slabs. The experimental results include the crack pattern characteristics, records of punching load capacity versus vertical deflection, strain in tensile reinforcement mesh, strains and contribution of shear head sections.

### CRACK PATTERN CHARACTERISTIC

The mode of failure was brittle since the failure occurred in concrete due to punching shear. This doesn't prevent some minor flexural cracks. This can be interpreted as after punching failure of specimen, the specimen returns to bear some flexural stresses which in turn leads to flexural cracks. The crack pattern and mode of failure at the top and bottom of the flat slab specimen (S-L1-2) plotted in Figures (9), (10).

### LOAD-VERTICAL DEFLECTION RELATION

The deflections were measured by LVDT gauges at equally spaced locations. The deflection profiles showed the deformational response to the application of load along the slab width. Figures (11), (12) and (13) show the load- vertical deflection curves for specimens with column aspect ratio (1:1), (2:1) and (4:1).

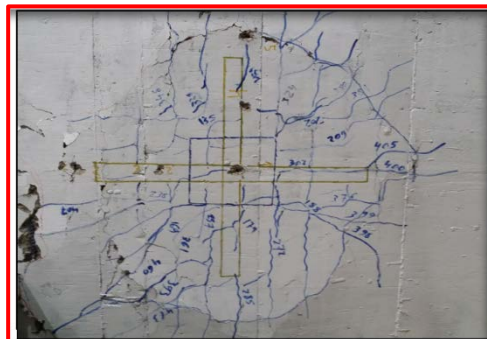


Figure (9): Crack pattern at Tension Zone of Specimen with Shear Heads.





Figure (10): Crack Pattern at Compression Zone of Specimen with Shear Heads.

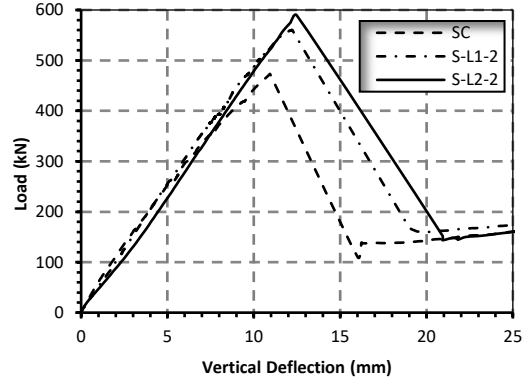


Figure (11): The Load-vertical deflection curve for Specimens with Column Aspect Ratio (1:1)

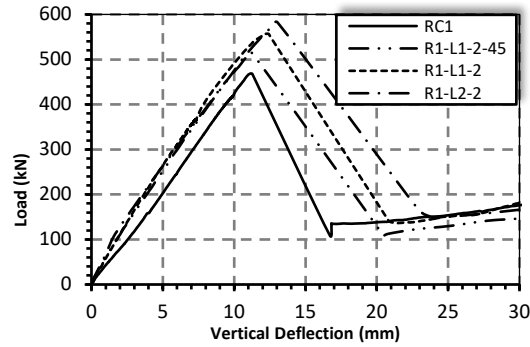


Figure (12): The Load-vertical deflection curve for Specimens with Column Aspect Ratio (2:1)

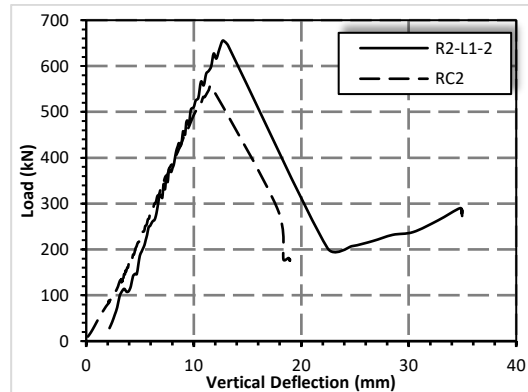


Figure (13): The Load-vertical deflection curve for Specimens with Column Aspect Ratio (4:1)

#### RECORDED STRAIN IN TENSION STEEL AT COLUMN FACE

The maximum tension steel strain was recorded at column face for all tested specimens. The recorded tension steel strain for specimens with steel shear heads reached yielding point. This recorded bottom steel strain confirms the (punching/flexural) failure occurrence compared with the control specimens. For example,

Figure (14) shows the load versus recorded strain in tension steel at column face for specimens with column aspect ratio (1:1).

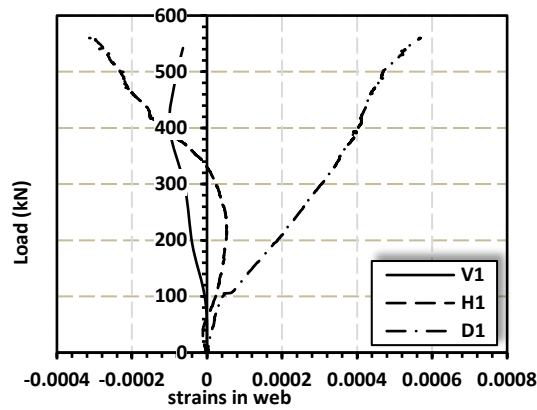


Figure (15): Applied Load versus Strains of Web at  $d/2$  for Specimen (S-L1-2).

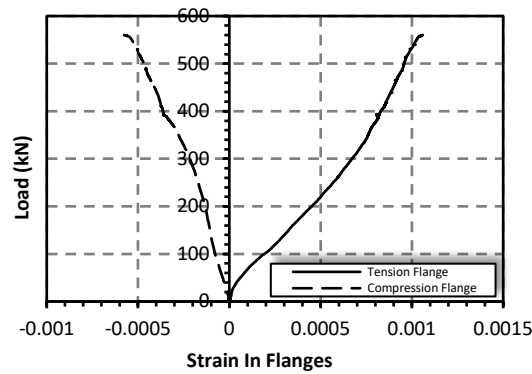


Figure (16): Applied Load versus Strains of Web at  $d/2$  for Specimen (S-L1-2).

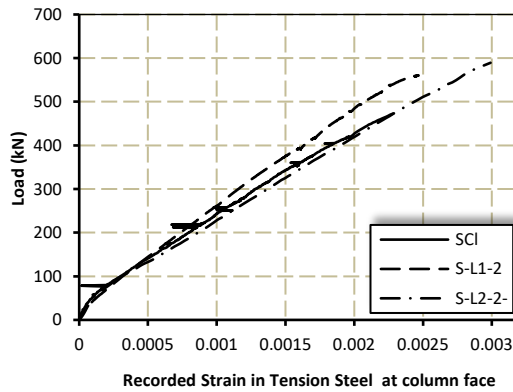


Figure (14): The Load versus Recorded Strain in Tension Steel at Column Face for Specimens with Column Aspect Ratio (1:1).

#### RECORDED STRAINS AND CONTRIBUTION OF SHEAR HEADS

Recorded normal strains in web, top and bottom flange at distances  $d/2$ ,  $d$  and  $0.75(L_v - c/2)$  from column face were recorded for all specimens with shear heads. For example, Figures (15) and (16) show results of groups from recorded normal strains at distance  $d/2$  for specimen (S-L1-2) in web and flange respectively.



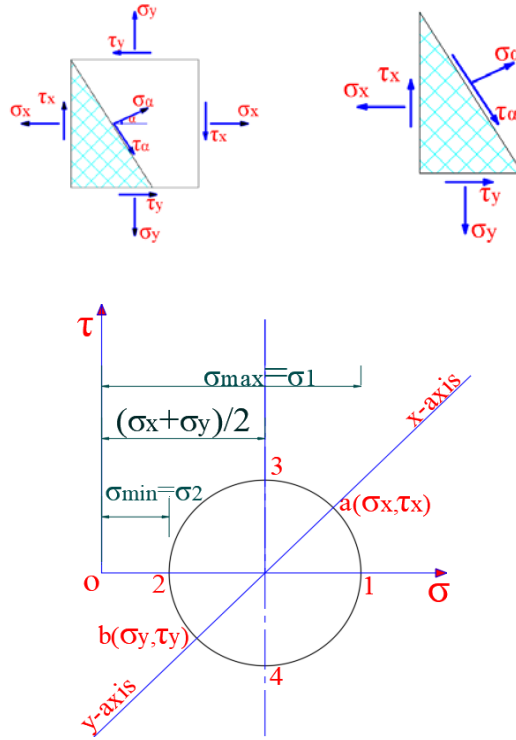


Figure (17): Stress Transformation : Graphical Illustration [Mohr Circle].

The following equations explain the mechanism of calculating contribution of shear heads in punching shear as shown in Figure (17).

$$\sigma_{\alpha} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha - \tau_x \sin 2\alpha \quad (19)$$

$$V = 4\tau_x \cdot A_{web} \quad (20)$$

The number "4" indicated for four leg sides of shear heads. For example, contribution of shear heads for specimens with shear heads in which connected by rectangle column shows in Figure (18).

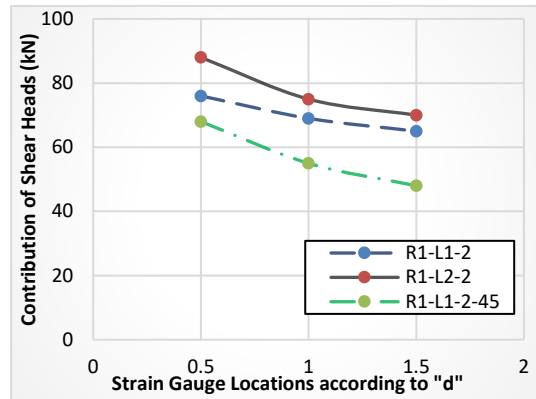


Figure (18): Contribution of Steel Shear Heads for Specimens which Connected by Rectangle Columns.

Two nodes are required for this element. Each node has three degrees of freedom, translation in the nodal x, y, and z direction. The element is also capable of plastic deformation.

**SHELL 181**, is suitable for analyzing thin to moderate-thick element with six degrees of freedom at each node and valid for simulating steel shear head sections.

**SOLID 45** element used for simulate steel plates at the supports for the column. This element has eight nodes with three degrees of freedom.

Finite element modelling for flat slab specimen with shear heads shown in Figure (19).

Material properties for concrete is well described in finite element program. There are multiple parts of the material model for the concrete element as mentioned by Badawy M.M. et. al. for the concrete element, this material model refers to solid 65 elements. And it is defined as linear isotropic for the elastic zone and multilinear isotropic for the plastic zone of the concrete. The steel for the finite element models was assumed to be an elastic-perfectly plastic material and identical in tension and compression. The material modeled as a linear isotropic element with modulus of elasticity for the steel ( $E_s$ ), and poison's ratio (PRXY) for lead plates and supports. The material model refers to the SOLID 45 element.

#### NUMERICAL ANALYSIS PROGRAM

Performing a numerical and analytical study then compared with additional experimental investigation of flat slabs with shear heads to enhance the understanding of the behavior and the mechanism of punching failure.

All details of modelling have been described using finite element program (ANSYS 12.0).

**Solid65**, an eight-node solid element, is used to model the concrete. The solid element has eight nodes with three degrees of freedom at each node-translations in the nodal x, y, and z directions. The element is capable of plastic deformation, cracking in three orthogonal directions, and crushing.

**Link8**, truss element is used to model the steel reinforcement.

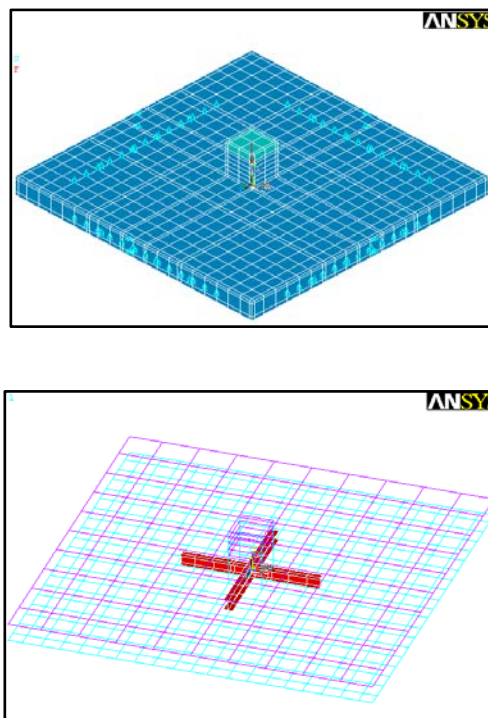


Figure (19): Finite Element Modelling for Flat Slab Specimen with Shear Heads.

Table (1): Concrete Mix Proportions.

Material	Dolomite(kg)	Sand(kg)	Cement(kg)	Water(Litre/kg)
Mix Proportion (Kg/m <sup>3</sup> )	898	863	384	230

Table (2): The yield Strength, Ultimate Strength and Elongation.

Material	Nominal Diameter (mm)	Yield Strength (N/mm <sup>2</sup> )	Yield Strain	Ultimate Strength (N/mm <sup>2</sup> )
Steel (40/60)	10	440	0.002	624
Steel (40/60)	18	490	0.0025	668

Table (3): The yield strength, ultimate strength and elongation of steel in which shear heads made from.

Material	Yield Strength (N/mm <sup>2</sup> )	Yield Strain	Ultimate Strength (N/mm <sup>2</sup> )
Steel 37	300	0.0015	440

Table (4): The yield strength, ultimate strength and elongation of steel in which shear heads made from.

Group	Specimen	Column Aspect Ratio	Column Dimensions	Shear Head Length	Remarks
<b>Group (A)</b>	SC	1	220*220	without	Control specimen
	S-L1-2	1	220*220	1.75 h	Cut end angle=90°
	S-L2-2	1	220*220	2.25 h	Cut end angle=90°
<b>Group (B)</b>	RC1	2	150*300	without	Control specimen
	R1-L1-2	2	150*300	1.75 h	Cut end angle=90°
	R1-L1-2-45	2	150*300	1.75 h	Cut end angle=45°
	R1-L2-2	2	150*300	2.25 h	Cut end angle=90°
<b>Group(C)</b>	RC2	4	110*440	without	Control specimen
	R2-L1-2	4	110*440	1.75 h	Cut end angle=90°

#### PARAMETRIC STUDY

A wide range of parametric study to investigate influence of using of embedded steel shear head sections on reinforced concrete flat slabs under punching shear forces. This parametric study suggested to obtain specific guidelines to help structural engineers who deal with this problem. The first investigated parametric study was as follow:

- 1- Different column aspect ratio (a/b).
- 2- Length of steel shear head section from column face (L).
- 3- Different arrangement of steel shear head sections (A).

In order to investigate the effect of the previous parameters on the behavior of specimens, the specimens are classified in the following flow chart, each with a particular parameter. These groups could be described as follows in Figure (20).

Changing compressive strength of concrete ( $f_{cu}$ ) from 25 MPa to 40 MPa with increment 5 MPa taking in to consideration. This is being the second investigated parametric study which shown in Figure (21). Also another parameter taking into consideration, this parameter was studying the effect of using shear studs welded in shear heads at compression zone (ST). This parameter was made for the specimen which has a column aspect ratio (a/b) equal (1.00) and choice (2.00d) as a length for shear head from column face.

Arrangement (1,1) means using one shear head leg for each direction, Arrangement (2,1) means using two legs of shear heads perpendicular to the long direction of the column, More over one leg of shear head perpendicular to short direction. Arrangement (2,0) indicated for using only two shear head legs perpendicular to the long direction. Figures [22-25] show modeling and shear stresses of shear heads for specimens with different arrangement (2,1) and (2,0).

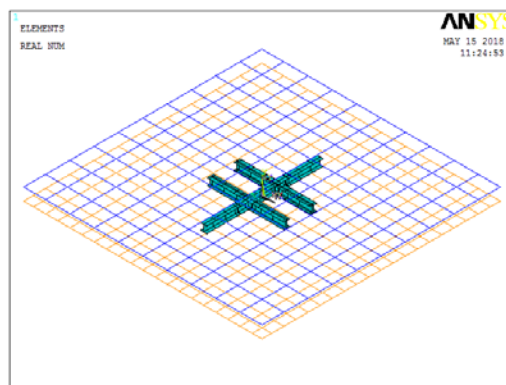


Figure (22): Finite Element Modelling of Specimen R4:1-2.00d-2,1

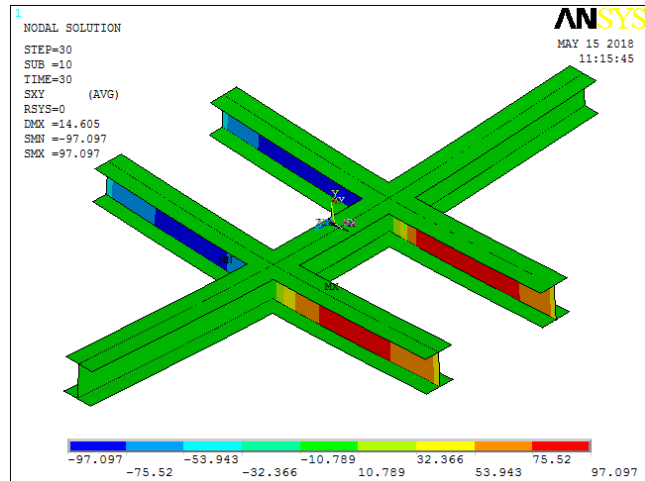


Figure (23): Shear Stresses in Short Legs for Specimen R4:1-2.00d-2,1

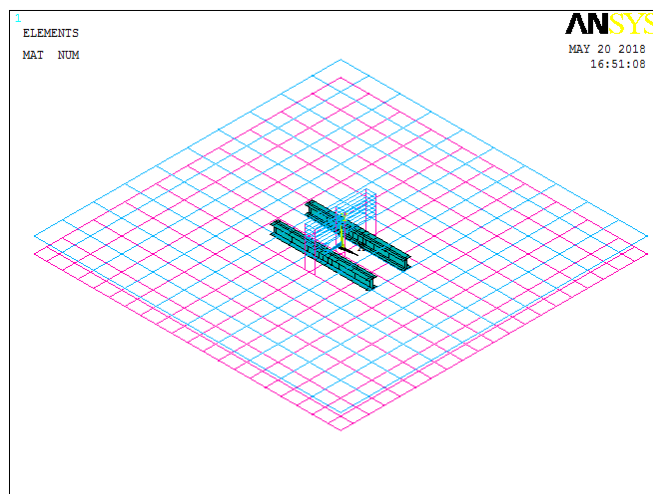


Figure (24): Finite Element Modelling of Specimen R4:1-2.00d-2,0

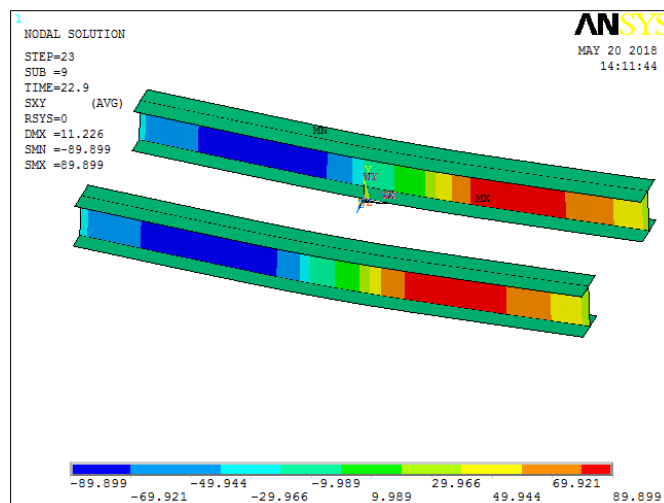


Figure (25): Shear Stresses in Legs for Specimen R4:1-2.00d-2,0

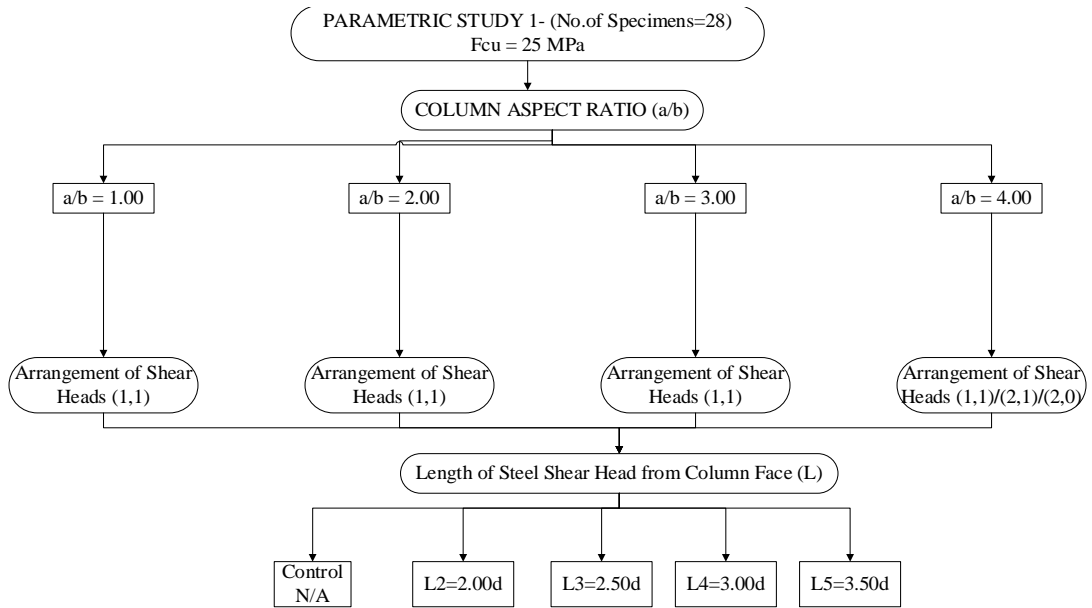


Figure (20): Parametric Study (I).

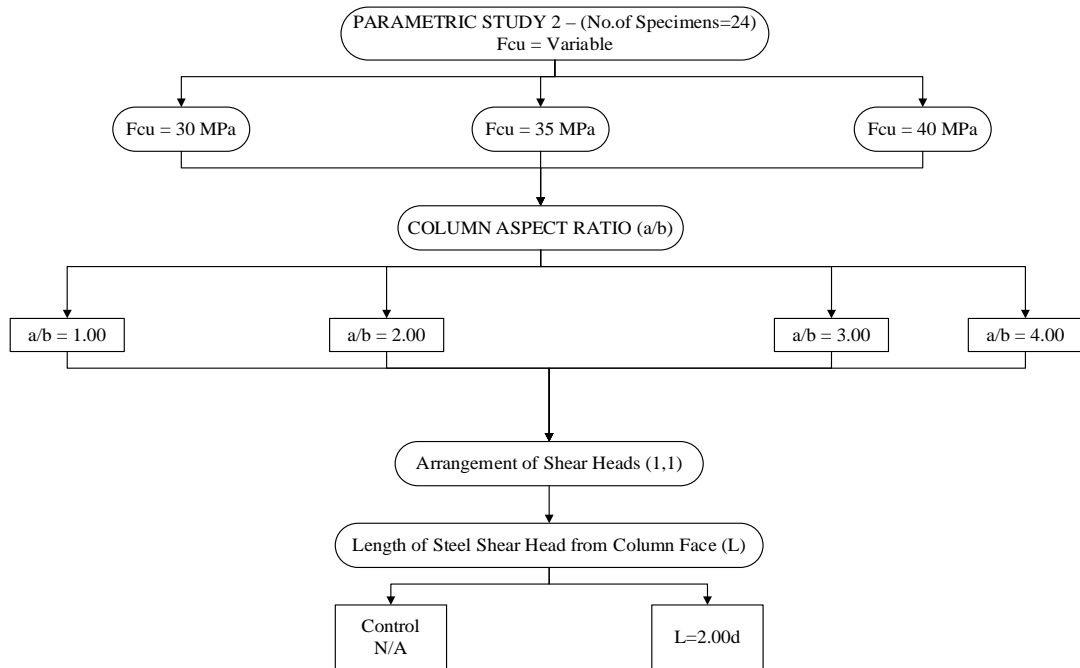


Figure (21): Parametric Study (II).

Figure (26) shows comparison for specimens of column aspect ratio (4:1) with different arrangement [1,1-2,1-2,0]. A group of new specimens with different main longitudinal steel reinforcement had been made. Main longitudinal steel reinforcement changed to be  $\Phi 22@100\text{mm}$  ( $\rho=2.5$ ) instead of  $\Phi 18@100\text{mm}$  ( $\rho=1.7$ ). Eight specimens were analyzed using "ANSYS" program. Four different column aspect ratio were used (1:1, 2:1, 3:1 & 4:1) with two different lengths of shear heads (2.00d, 2.5d) from column face. Figure (27) shows results of specimens with different main longitudinal steel reinforcement ratio [ $\rho = 1.7\%$  &  $\rho = 2.5\%$ ].

From the observation of the crack pattern at the top of all most specimens. It was found that the cracks began almost beyond the column face. So the cracks created a pathway in the concrete until reaching the beginning of shear heads. From here came the idea of making composite shear reinforcement system between shear heads and studs. Shear studs were welded with shear heads at compression zone of the slab. This is in order to try to increase the punching circumference at compression zone of the slab. Specimens with column

aspect ratio (1:1) were chosen for this study and have the same compressive strength of concrete (25MPa). This comparison has been made for different lengths of shear heads which ranged from (2.00d) to (3.50d).

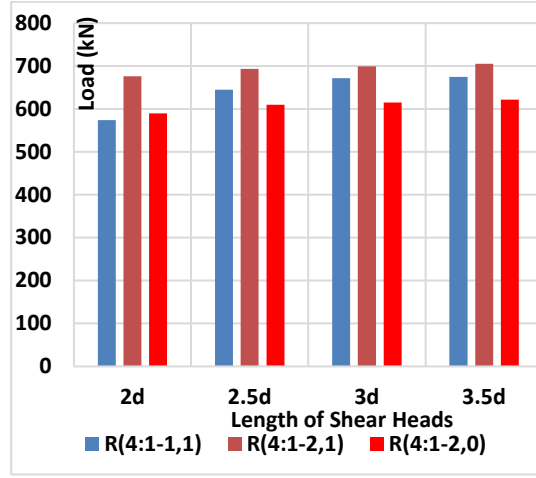


Figure (26): Comparison for Specimens of Column Aspect Ratio (4:1) with Different Arrangement [1,1-2,1-2,0].

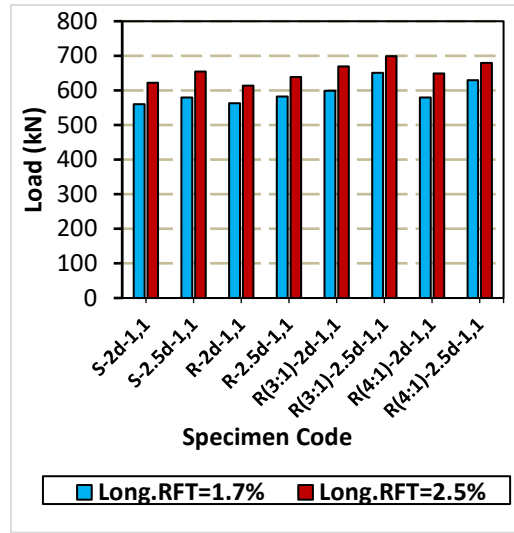


Figure (27): Bar Chart of Specimens with Different Main Longitudinal Steel Reinforcement Ratio [ $\rho = 1.7\%$  &  $\rho = 2.5\%$ ]

#### PROPOSED EQUATION

Figure (28) shows philosophy of crack path which was derived from experimental program. The proposed equation was constructed on two perimeters, small perimeter at compression zone and large perimeter at tension zone.

These steps could be followed to obtain the parameters in the proposed equation:

$$b_{o1} = 2[(a + a_1) + (b + a_1)] \quad (19)$$

$$a_1 + a_2 = d \quad (20)$$

By regression analysis:

$$A = a + 0.48 \times d \quad (21)$$

$$B = b + 1.45d \quad (22)$$

$$L_x = L_{sh(x)} + \frac{a_2}{2} \quad (23)$$

$$L_y = L_{sh(y)} + \frac{a_2}{2} \quad (24)$$

$$L' = \sqrt{\left(L_x - \frac{A}{2}\right)^2 + \left(L_y - \frac{B}{2}\right)^2} \quad (25)$$

$$b_{o2} = 4L' + 2A + 2B \quad (26)$$

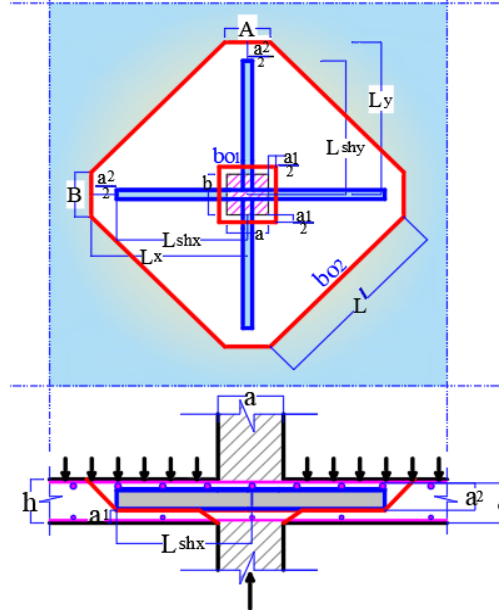


Figure (28): Crack Path Model for Flat Slabs with Shear Heads.

a, b are short and long sides of column respectively.

The proposed design formula for punching load capacity could be expressed as follows

$$V = 0.63 \left(0.5 + \frac{a}{b}\right) \sqrt{f_{cu}} \times b_{o1} \times a_1 + 0.26 \times \sqrt{f_{cu}} \times b_{o2} \times a_2 \quad (27)$$

Where:

$$0.5 + \frac{a}{b} \leq 1.00$$

V: Ultimate Punching Load (N).

$f_{cu}$ : Standard Cubic Strength of Concrete (MPa).

$b_{o1}$ : Punching Perimeter at Compression Zone (mm).

$a_1$ : Distance from Compression Fiber of Concrete up to Compression Fiber of Shear Heads (mm).

$b_{o2}$ : Punching Perimeter at Tension Zone (mm).

$a_2$ : Distance from Compression Fiber of Shear Heads up to c.g of Tension Reinforcement.

Figure (29) shows comparison between experimental, numerical and proposed design equation. This proposed design equation is conservative because of all expected results from proposed equation are less than the results of the experimental and numerical analysis. This is due to the brittle failure of flat slabs under punching shear.

Figure (30) shows crack path model where suggested by Sherif and Dilger (1995). Failure perimeter is proposed outside of shear reinforcement zone at distance  $d/2$  from shear head ends.



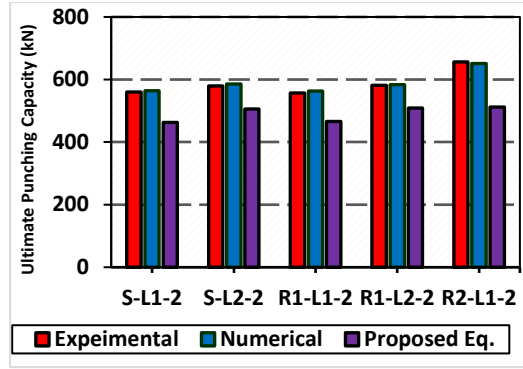


Figure (29): Comparison between Experimental, Numerical and Proposed Design Equations.

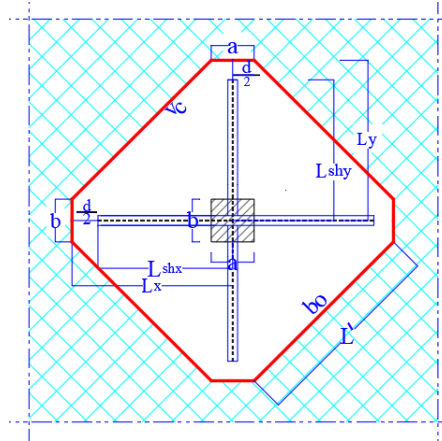


Figure (30): Crack Path Model Suggested by Sherif and Dilger (1995).

#### COMPARISON USING "SHERIF AND DILGER" (1995) FORMULA

Sherif and Dilger (1995) recommended that the shear stress resistance be expressed as a function of the distance from column face and percentage of main longitudinal reinforcement.

$$v_c = 0.7(100\rho f'_c)^{\frac{1}{3}} \left(0.1 + \frac{1}{\alpha_o^{0.63}}\right) \leq 0.7(100\rho f'_c)^{\frac{1}{3}} \quad (28)$$

Where:

$v_c$ : punching shear stress resistance at a critical section.

$\rho$ : percentage of main longitudinal reinforcement.

$\alpha_o$ : Distance of shear head from column face divided by depth.

$f'_c$ : Cylindrical compressive strength of concrete.

Because of the decrease of shear stress resistance below  $0.2\sqrt{f'_c}$  (value specified by CSA) for  $\alpha_o > 5$ , it is recommended that  $\alpha_o$  should not exceed 5, which means that shear reinforcement should not be extended beyond a distance of  $4.5d$ .

Figure (31) shows shear stress resistance as a function of distance from column face divided by depth "d".

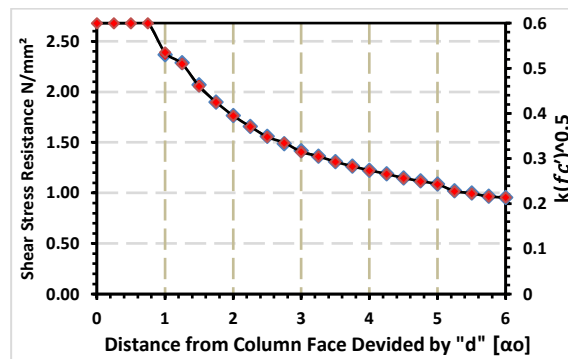


Figure (31): Shear Stress Resistance as a function of distance from column face divided by depth "d".

## CONCLUSION

Flat slabs with shear heads enhanced the mode of failure to be semi brittle failure. The European code (EC2-2004) is the closest code for finding concrete contribution in punching shear resistance. The new proposed formula has an original contribution and a good agreement in prediction of punching shear capacity of flat slabs with shear heads. Shear heads with length equal to  $2d$ ,  $2.5d$ ,  $3d$ ,  $3.5d$  enhanced punching shear capacity of flat slabs by 18, 22, 33 and 31.6 respectively for flat slabs with column aspect ratio 1:1, 2:1, 3:1 and 4:1. Punching shear capacity of flat slabs enhanced by 10, 19 and 27% when compressive strength of concrete changed to be 30, 35 and 40 MPa respectively.

Rates of improvement in punching shear force for specimens with the new arrangement (2,1) higher by (16.5, 10, 5 and 4%) compared to specimens with arrangement (1,1) at the same used lengths of shear heads ( $2d$ ,  $2.5d$ ,  $3d$  and  $3.5d$ ) respectively. The new arrangement of shear heads (2,0) showed its inability to increase the punching shear resistance of flat slabs with column aspect ratio (4:1) compared to the basic arrangement (1,1) at the same used lengths of shear heads ( $2d$ ,  $2.5d$ ,  $3d$  and  $3.5d$ ). To avoid slippage of steel shear heads in flat slabs, Minimum length of ( $2.00d$ ) from column face is the preferred length for shear heads. Maximum optimum length of shear heads for flat slabs with column aspect ratio (1:1), (2:1), (3:1) and (4:1) is equal to ( $3.00d$ ) from column face.

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