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Auto-Regression Auctions

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Abstract
This paper proposes two auto-regression auction formats namely, simple autoregression and multiple autoregression auctions. Both auto-regression auctions use the valuable information available in both past auctions and current auction to predict the start price, recommended price and reserve price of the current auction's object which allow auctioneers to maximize the current auction's revenue. In simple autoregression auctions ($SARA$), an autoregression moving average model $ARMA(p,q)$ is used to predict the start price of the current auction object based on a time series of start bids placed in past auctions, and predict the recommended object price of the current auction based on the time series of bids placed by all bidders during the current auction. The reserve price that can be accepted by the seller of the object in the current auction is predicted using $ARMA(p,q)$ based on the highest bids placed by the winners of past auctions. In multiple auto-regression auctions ($MARA$), the auction runs in multiple rounds where the start price of the first round in the current auction is predicted based on a time series of start bids placed in past auctions using $ARMA(p,q)$ model. The temporary recommended price of the auction object in each round of the current auction is predicted using $ARMA(p,q)$ model based on a time series of all bids placed in the same round during the current auction. The temporary reserve price of the auction object in each round of the current auction is calculated based on a time series of bids placed in the same round of past auctions. The start price of each round is based on the reserve price of the auction object in the previous round and on a time series of past start bids in the same round of past auctions. At the end of the current auction whether it is SARA or MARA, the winner with the highest bid will pay the average of the predicted final recommended price and predicted final reserve price of the current auction object if the final recommended price is within the interval of the final reserve price and seller price.
Keywords: Auctions, Bids, Time Series, Auto-regression Models, Moving Average, Open Auctions, Closed Auctions

1. Introduction
Auctions are an important tool to determine a fair market value through competitive bidding and allocation of commodities and financial assets to individuals, organizations, and firms (Robert A. Feldman et al., 1993). Selling an item for a fair price is the main objective of auctions. The determination of an auction winner in conventional auctions such as the English auction is very simple as it only requires the identification of the bidder with highest bid price. The motivation of this paper is that the market value determined by the highest bid price may not be the true value of the auctioned object as the true valuations of buyers are not known. Furthermore, open auction formats allow for the possibility that buyers collude. (Graham et al.,1987) argue that such collusion is facilitated in open auctions, where buyers can directly inspect one another's behaviour. Also, in closed auctions buyers can collude by either making deals or obtain secret information about other bidders. In other words, open auctions and closed auctions do not allow to determine the fair market price to be paid by the winner of the auctioned object as the true valuations of the buyers are unknown and the private information that leaked to the buyers are also unknown. Hence, prediction of the fair market price is required based on the past bids placed by all buyers participated in the auction. Based on the above unknowns, the main objective of this paper is to introduce a new auction format that we believe it determines a fair market price of the auctioned object. It is worth noting that the theoretical model of regression auctions presented in this paper was not experimented in the real world and thus the difficulties of

applying this model in real life still unknown. It is well known that there are many types of auctions namely, the English auction is an open outcry ascending auction where bids start from lower price and reaches the highest price. Bidders that have an interest in the auctioned item start placing bids with the auctioneer accepting higher bids as they come.

The English auction is fully transparent as information is shared between bidders in real time. Each bidder's private information about the common value of the auctioned item is a valuable information to the other bidders and is disclosed during the public bidding process (Milgrom, Paul, 2004). The English auction proceeds as follows:

- The auctioneer starts the auction by an opening bid price. If no bidder accepts this price the auctioneer either lowers the starting price or allow bidders to bid at lower price. This must be arranged with the owner of the auctioned item.
- Bidders compete by placing bids in a highly increasing manner.
- The highest bid price at any moment is the standing bid until it is replaced by a higher bid.
- If no rival bidder places a bid with a higher price than the standing bid within the time allowed by the auctioneer, the bidder with the standing bid is declared as a winner.

The Dutch Auction is also an open outcry descending price auction or clock auction where the bidding starts from the highest price and reaches the lower price but not less than the predetermine reserved price (Li, Zhen, 2011). The strategy in Dutch auction the auctioneer starts with a high price and gradually declines this price. Each bidder watches the price as it declines until it reaches a point where a bidder or a rival bidder claim by shouting “it is mine” and the auction ends. The Vickery auction is known as Second price sealed-bid auction. None of the bidders know what the other is offering. The bidder with the highest price wins, but only pays the value of the next highest bid (Mierendorff, Konrad, 2013). The most important feature of these auctions is that each bidder's winning strategy is to bid according to their true valuation of the item; this auction mechanism is thus incentive compatible. It is also a Pareto efficient allocation mechanism (Rothkopf et al, 1990). In the First-Price auction all bidders simultaneously submit sealed bids so that no bidder knows the bid of any other participant. The highest bidder pays the price that was submitted (Krishna et al., 2002). In the second price auctions the highest bidder wins but only pays the price equal to the second-highest bid. In double auction multiple sellers ask their prices and multiple buyers submit their bids. The auctioneer chooses a starting price p and all sellers ask less than price p sell and all buyers bid than price p buy. The Japanese auction is also called ascending clock auction is a dynamic auction format. It proceeds in the following way (Milgrom et al., 2014):

- An initial price is displayed. This is usually a low price - it may be either 0 or the seller's reserve price.
- All buyers that are interested in buying the item at the displayed price enter the auction arena.
- The displayed price increases continuously, or by small discrete steps (e.g. one cent per second).
- Each buyer may exit the arena at any moment.
- No exiting buyer is allowed to re-enter the arena.
- When a single buyer remains in the arena, the auction stops. The remaining buyer wins the item and pays the displayed price.

In a Scottish auction (or also time-interval auction), all bidding should be completed within a certain time interval (Hultmark et al., 2002). This ruling provides the bidders an appropriate amount of time for consideration. Speed is not important in this type of auction. In the simultaneous multiple round auctions (SAR) all bidders bid on all items in the same time, the starting price is low and the winner pays the highest price. This auction runs in rounds and after each round of bids the highest bid for each item is announced and a new round begins with new information (Bachler, M et al., 2017).

2. Related Work

2.1. Quantile regression methods for first-price auctions

In this paper a quantile-regression model for first-price auctions with symmetric risk-neutral bidders under the independent private-value paradigm is proposed. Quantile regression for the bids can be generated from private-value quantile regression and its derivative with respect to the quantile level (Nathalie Gimenes, 2022).

2.2. Auction optimization using regression trees and linear models as integer programs

This paper solves the problem of finding the optimal ordering of sequential auctions using machine learning techniques. Two types of optimization methods are proposed namely, black-box best-first search approach, and a novel white-box approach that maps learned regression models to integer linear programs (ILP), which can then be solved by any ILP-solver It also uses regression models based on data from historical auctions to predict the expected value of orderings for new auctions (Sicco Verwer et al., 2017).

3. Motivation

All the above auction formats do not use regression models to help auctioneers tune auction parameters to maximize the auction revenue. The information available from historic auctions and bids placed by bidders in the current auction are used by an autoregression moving average model $ARMA(p,q)$to improve auctions revenue. The auction optimization method that uses regression trees and linear models optimizes the ordering of new sequential auctions based on past historic auctions which do not tune the auction's parameters for the auctioneer to maximize the auction's revenue.

4. Autoregressive Model

An auto-regression model $AR(p)$ determines the value of a process at an arbitrary time i using a linear combination of the p-last values (Ullrich, T., 2021):

$$y_i = \sum_{j=1}^{p} \emptyset_j y_{i-j} + \varepsilon_i \quad (1)$$

The value of p indicates the order of the AR model, the weights $\emptyset_i$of the linear combination are the model parameters and ε_t is random error assumed to be $N(0,1)$.

5. Moving Average

A q-order moving average (MA) process**,** denoted $MA(q)$ takes the form (Charles Zaiontz, 2023):

$$y_i = \mu + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j} + \varepsilon_i \quad (2)$$

Where i is an arbitrary time such that the value of y at time i is a linear combination of past errors. These error terms are assumed to be independently distributed where $\varepsilon_i \sim N(0,\sigma^2)$.

6. Autoregressive Moving Average

An autoregressive moving average (ARMA) process consists of both autoregressive and moving average terms. If the process has terms from both $AR(p)$ and MA(q) process, then the process is called $ARMA$(*p, q*) and can be expressed as follows:

$$y_i = \varepsilon_i + \sum_{j=1}^{p} \emptyset_j y_{i-j} + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j} \quad (3)$$

To estimate the model parameters $\emptyset_j, \forall\, j = 1,2 \ldots\ldots, p$ and $\theta_j\,, \forall\, j = 1,2 \ldots\ldots, q$ we use the least squares method (Bowler, Ben et al., 2020).

Let:

$$\varepsilon_i = y_i - \left(\sum_{j=1}^{p} \emptyset_j y_{i-j} + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j}\right) \quad (4)$$

Minimise:

$$\sum_{i=p+1}^{n} {\varepsilon_i}^2 = \sum_{i=p+1}^{n} \left(y_i - \left(\sum_{j=1}^{p} \emptyset_j y_{i-j} + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j}\right)\right)^2 \quad (5)$$

Let X be $(n-p) \times (p+q)$ matrix such that the i^{th} row is:

$$[\emptyset_1 y_{i-1} + \emptyset_2 y_{i-2} + \cdots + \emptyset_p y_{i-p} + \theta_1 \varepsilon_{i-1} + \theta_2 \varepsilon_{i-2} + \cdots + \theta_q \varepsilon_{i-q}]$$

Let Y be $n-p\ \times\ 1$ column vector such that:

$$Y = [y_{p+1} y_{p+2} \ldots . y_n]^T \quad (6)$$

Let φ be $p\ \times 1$ coulmn vector such that:

$$\varphi = [\emptyset_1 \emptyset_2 \dots \dots . \emptyset_p \theta_1 \theta_2 \dots \dots \theta_q]^T \quad (7)$$

Then the $ARMA(p, q)$ can be represented as follows:

$$Y = X\varphi + \varepsilon \quad (8)$$

Therefore:

$$\varphi = (X^T X)^{-1} X^T Y \quad (9)$$

7. Simple Auto-Regression Auction (SARA)

Setting the starting price is critical to attract buyers and generate revenue from an auction. Too high starting price limits the buyers' interest. How to set the start price of an auction object? How to set the reserved price accepted by the seller? To set the starting price some auctions use the 30% fair market value as a starting price for bids. Some auctioneers find similar objects and compare prices across websites and in stores if possible. This helps the auctioneer determines what the item typically sells for. The regression auction format $ARMA(p, q)$ predicts the starting price of the auction object based on a time series of start bids placed in past auctions. Also, $ARMA(p, q)$ predicts the reserved price $\hat{r}$ of the auction object based on the past highest bids placed by the winners in past auctions. The recommended price $\hat{b}$ of the current auction object is predicted based on all bids placed by all bidders during the current auction. The prediction of the reserved price protects the seller from setting a price lower than the true value of the object. The auction proceeds as follows:

- Buyers compete by placing higher bids based on the predicted start price $\hat{v}$ using $ARMA(p, q)$.
- At the end of the auction the recommended price $\hat{b}$ is predicted using $ARMA(p, q)$ based on a time series of all bids placed by all bidders including the highest bid placed by the winner.
- At the end of an auction, the reserve price of the object $\hat{r}$ to be accepted by the seller is predicted using $ARMA(p, q)$ based on a time series of past highest bids placed by the winners of past auctions.
- The winner with the highest bid in the current auction will pay the object price p such that:

$$p = \frac{\hat{r} + \hat{b}}{2} \quad (10)$$

where $\hat{b} \in [\hat{r}, s]$ where s is the seller price.

7.1. Start Price Prediction

An autoregression moving average $ARMA(p.q)$ model uses a series of start bids that were placed in past auctions to predict the starting price b for the current auction such that:

$$v = \varepsilon_i + \sum_{j=1}^{p} \emptyset_j v_{i-j} + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j} \quad (11)$$

Let:

$$\hat{v} = \hat{v}_i = \sum_{j=1}^{p} \emptyset_j v_{i-j} + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j} \quad (12)$$

Where:

v: is the true value of the start price for the current auction.

$\hat{v}$: is the estimated value of the start price for the current auction.

v_{i-j}: is the start bid price placed at the j^{th}past auction.

7.2. Recommended Object Price Prediction

The autoregression moving average $ARMA(p.q)$ model represents the times series of all bids placed by the bidders of the current auction to predict the recommended price $\hat{b}$ of the auction object such that:

$$b = \varepsilon_i + \sum_{j=1}^{p} \emptyset_j b_{i-j} + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j} \quad (13)$$

Let:

$$\hat{b} = \sum_{j=1}^{p} \emptyset_j b_{i-j} + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j} \quad (14)$$

7.3. Reserved Price Prediction

The autoregression moving average $ARMA(p.q)$ model uses a series of highest bids that were placed by the winners of past auctions to predict the reserve price $\hat{r}$ to be used by the auctioneer such that:

$$r = \varepsilon_i + \sum_{j=1}^{p} \emptyset_j r_{i-j} + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j} \quad (15)$$

Let:

$$\hat{r} = \hat{r}_i = \sum_{j=1}^{p} \emptyset_j r_{i-j} + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j} \quad (16)$$

Where:

r: is the true reserve price of the current auction object.

$\hat{r}$: is the predicted reserve price of the current auction object.

r_{i-j}: is the highest bid price placed by the winner of the j^{th}past auction.

7.4. Game Theoretic Analysis

Simple regression auctions can be analysed as a game where the players are buyers and sellers. Sellers ask for bids and buyers place their bids. The interesting problem is to find Nash equilibrium.

Assume:

- The seller asks price s.
- The auctioneer predicts reserve price $\hat{r}$.
- The auctioneer predicts recommended price $\hat{b}$.
- The true value of the buyer is B.
- The true value of the seller is S.

The auctioneer sets the following rules:

- If $\hat{b} < \hat{r}$ then no trade.
- If $\hat{r} \leq \hat{b} < s$ then the buyer pays $p = \frac{(\hat{b}+\hat{r})}{2}$.
- If $\hat{b} \geq s$ then the buyer pays $p = \frac{(\hat{b}+s)}{2}$.

The utility of the seller is:

- If $s > \hat{b}$ then utility is 0.
- If $s \leq \hat{b}$ then utility $p - S$.

The utility of the buyer is:

- If $\hat{b} > s$ then utility is 0.
- If $\hat{b} \leq s$ then utility is $B - p$.

The Nash equilibrium exists with

$\hat{b} = \hat{r} = s \in [B, S]$ otherwise, there will be no equilibrium.

8. Multiple Auto-Regression Auctions (MARA)

Sometimes it is hard to determine the start price of an auction. For example, it is hard to determine the starting price of vacation packages, event tickets, and one-of-a-kind memorabilia. So, a more sophisticated regression auction format is required. The multiple round regression auction ($MARA$) works in rounds and predicts for each round a start price, temporary object price and temporary reserve object price. In the final round, $MARA$ predicts the final recommended object price and final reserve object price of the current auction. The winner with the highest bid will pay the average of the final recommended object price and final reserve object price if the highest bid of the winner is within the interval of final reserve object price and seller price. The $MARA$ works as follows:

- The auctioneer creates a bid time series of the first round of past auctions using the start bids $v_{i-1,j}^{1}, v_{i-2,j}^{1} \dots, v_{i-p,j}^{1}$ placed by multiple buyers at multiple times such that:

$$\hat{v}_{ij}^{1} = \sum_{h=1}^{p} \emptyset_h v_{i-h,j}^{1} + \sum_{h=1}^{q} \theta_h \varepsilon_{i-h,j}^{1}, \forall j = 1,2, \dots \dots \dots .., m \quad (17)$$

Where:

$v_{i-h,j}^{1}$: is the start price of the first round at time h of the j^{th}past auction.

- The auctioneer predicts the start price $\hat{v}_{i,j}^{1}$ of the first round of the j^{th} past auction using $ARMA(p,q)$.
- The auctioneer calculates the predicted average start price $\bar{v}^{1}$ of the first round of the current auction using the m past auctions as follows:

$$\bar{v}^{1} = \bar{v}_{i}^{1} = \frac{1}{m} \sum_{j=1}^{m} \hat{v}_{i,j}^{1} \quad (18)$$

- The buyers place their bids based on the predicted average start price $\bar{v}^{1}$.

At the end of the first round the auctioneer does the following:

- Create a bid time series of the first round of the current auction where $b_{i-1}^{1}, b_{i-2}^{1} \dots, b_{i-p}^{1}$ are the bids placed by multiple buyers at multiple times such that:

$$\hat{b}^{1} = \hat{b}_{i}^{1} = \sum_{j=1}^{p} \emptyset_j b_{i-j}^{1} + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j}^{1} \quad (19)$$

where: $b_{i-h}^{1}$is the bid in the first round of the current auction placed at time h.

- Use $ARMA(p,q)$ to predict the temporary recommended price $\hat{b}^{1}$.
- Create a bid time series of the first round of m past auctions where $r_{i-1,j}^{1}, r_{i-2,j}^{1} \dots, r_{i-p,j}^{1}$ are the bids placed by multiple buyers at multiple times in the j^{th} past auction such that:

$$\hat{r}_{ij}^{1} = \sum_{h=1}^{p} \emptyset_h r_{i-h,j}^{1} + \sum_{h=1}^{q} \theta_h \varepsilon_{i-h,j}^{1}, \forall j = 1,2, \dots \dots \dots .., m \quad (20)$$

$r_{i-h,j}^{1}$: is the temporary reserve price of the first round in the j^{th} past auction placed at time h.

- Use $ARMA(p,q)$ to predict the temporary reserve object price $\hat{r}_{i,j}^{1}$ of the first round in the current auction based on the j^{th} past auction.
- Calculate the average temporary reserve object price $\hat{r}^{1}$ of the first round in the current auction based on the m past auctions as follows:

$$\hat{r}^{1} = \frac{1}{m} \sum_{j=1}^{m} \hat{r}_{i,j}^{1} \quad (21)$$

- The predicted temporary reserve price $\hat{r}^{1}$ will be used as the start price for round 2.

For round $k = 2$ to $N - 1$:

The auctioneer uses the predicted temporary reserve price $\hat{r}^{k-1}$ to predict the start price $\hat{f}^{k}$ of the current auction object in round k as follows:

- Create a bid time series of the k^{th} round start prices $v_{i-1,j}^{k}, v_{i-2,j}^{k} \dots, v_{i-p,j}^{k}$ for the m past auctions using the bids placed by multiple buyers at multiple times such that:

$$\hat{v}_{ij}^{k} = \sum_{h=1}^{p} \emptyset_h v_{i-h,j}^{k} + \sum_{h=1}^{q} \theta_h \varepsilon_{i-h,j}^{k}, \forall j = 1,2, \dots \dots \dots .., m \quad (22)$$

$v_{i-h,j}^{k}$: is the starting price of the k^{th} round at time h of the j^{th}past auction.

- Use $ARMA(p,q)$ to predict the start price $\hat{v}_{i,j}^{k}$ of the k^{th} round of the current auction based on the bid time series of the k^{th} round of the j^{th} past auction.

- Calculate the average start price $\bar{v}^k$ of the k^{th} round of the current auction based on the k^{th} round of the m past auctions as follows:

$$\bar{v}^k = \bar{v}_i^k = \frac{1}{m}\sum_{j=1}^{m} \hat{v}_{i,j}^k \quad (23)$$

- Calculate the start price $\hat{f}^k$ of round k of the current auction as:

$$\hat{f}^k = \frac{\bar{v}^k + \hat{r}^{k-1}}{2} \quad (24)$$

- The buyers place their bids in round k based on the average predicted start price $\hat{f}^k$.

At the end of round k the auctioneer does:

- Create a bid time series of the k^{th} round of bids $b_{i-1}^k, b_{i-2}^k \dots, b_{i-p}^k$ that are placed by multiple buyers at multiple times of the k^{th} round in the current auction such that:

$$\hat{b}^k = \sum_{j=1}^{p} \emptyset_j b_{i-j}^k + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j}^k \quad (25)$$

b_{i-h}^k: is the bid placed in the k^{th} round of the current auction at the h^{th} time.

- Predict the temporary recommended object price $\hat{b}^k$ of round k in the current auction using $ARMA(p,q)$.
- Create a bid time series using the past bids $r_{i-1,j}^k, r_{i-2,j}^k \dots, r_{i-p,j}^k$ placed by the buyers of the k^{th} round at multiple times of the j^{th} past auction such that:

$$\hat{r}_{ij}^k = \sum_{h=1}^{p} \emptyset_h r_{i-h,j}^k + \sum_{h=1}^{q} \theta_h \varepsilon_{i-h,j}^k, \forall j = 1,2, \dots\dots\dots.., m \quad (26)$$

$r_{i-h,j}^k$: is the temporary reserve price of the current auction object in the k^{th} round of the j^{th} past auction.

- Use $ARMA(p,q)$ to predict the temporary reserve object price $\hat{r}_{i,j}^k$ of the k^{th} round in the current auction using based on the past bids placed in the j^{th} past auction.
- Calculate the average reserve object price $\hat{r}^k$ of the current auction object based on the m past auctions as follows:

$$\hat{r}^k = \frac{1}{m}\sum_{j=1}^{m} \hat{r}_{i,j}^k \quad (27)$$

- Let $\hat{r}^{k-1} = \hat{r}^k$. (28)
- Continue the For loop.

For last round N:

The auctioneer uses $\hat{r}^{N-1}$ to calculate the start price $\hat{f}^N$ for the last round N as follows:

- Create a bid time series for the N^{th} round of the past auctions using the start bids $v_{i-1,j}^N, v_{i-2,j}^N \dots, v_{i-p,j}^N$ placed by multiple buyers at multiple times such that:

$$\hat{v}_{ij}^N = \sum_{h=1}^{p} \emptyset_h v_{i-h,j}^N + \sum_{h=1}^{q} \theta_h \varepsilon_{i-h,j}^N, \forall j = 1,2, \dots\dots\dots.., m \quad (29)$$

$v_{i-h,j}^N$: is the start price of the N^{th} round at time h of the j^{th} past auction.

- Use $ARMA(p,q)$ to predict the start price $\hat{v}_{ij}^N$ of the N^{th} round of the current auction based on the j^{th} past auction.
- Calculate the average start price $\bar{v}^N$ of the N^{th} round of the m past auctions as follows:

$$\bar{v}^N = \bar{v}_i^N = \frac{1}{m}\sum_{j=1}^{m} v_{i,j}^N \quad (30)$$

- Calculate the start price $\hat{f}^N$ for round N of the current auction as follows:

$$\hat{f}^N = \frac{\hat{r}^{N-1} + \bar{v}^N}{2} \quad (31)$$

- Buyers place their bids based on the predicted start price $\hat{f}^N$.
- Create the bid time series of the N^{th} round of the current auction based on the bids $b_{i-1}^N, b_{i-2}^N \dots, b_{i-p}^N$ placed by multiple buyers at multiple times such that:

$$\hat{b}^N = \varepsilon_i^N + \sum_{j=1}^{p} \emptyset_j b_{i-j}^N + \sum_{j=1}^{q} \theta_j \varepsilon_{i-j}^N \quad (32)$$

b_{i-h}^{N}: is the bid price of the N^{th}round at h^{th}time in the current auction.

- Predict the final recommended object price $\hat{b}^N$ using $ARMA(p,q)$.
- Create a bid time series of the highest bids placed by multiple buyers in the N^{th} round of the j^{th}past auctions as follows:

$$r_{i-1,j}^{N}, r_{i-2,j}^{N}, \dots \dots \dots r_{i-p,j}^{N}$$

Therefore:

$$\hat{r}_{ij}^{N} = \sum_{h=1}^{p} \emptyset_h r_{i-h,j}^{N} + \sum_{h=1}^{q} \theta_h \varepsilon_{i-h,j}^{N}, \forall j = 1,2, \dots \dots \dots, m \quad (33)$$

$r_{i-h,j}^{N}$: is the highest bid price placed in the final round N at the h^{th} time of the j^{th} past auction.

- Predict the final reserve price $\hat{r}_{i,j}^{N}$ of the N^{th} round of the current auction based on the N^{th} round of the j^{th} auction using $ARMA(p,q)$.
- Calculate the average final reserve price $\hat{r}^N$in the last round N of the current auction based on the m past auctions as follows:

$$\hat{r}^N = \frac{1}{m} \sum_{j=1}^{m} \hat{r}_{i,j}^{N} \quad (34)$$

- The winner with the highest bid will pay the object price p:

$$p = \frac{\hat{r}^N + \hat{b}^N}{2} \quad (35)$$

where $\hat{b}^N \in [\hat{r}^N, s]$ where s is the seller price.

8.1. Game Theoretic Analysis

Multiple round regression auctions can be analysed as a game where the players are buyers and sellers. Sellers ask for bids and buyers place their bids. The interesting problem is to find Nash equilibrium.

Assume:

- The seller price is s.
- The true value of the buyer is B.
- The true value of the seller is S.

In round k:

- The temporary recommended object price is $\hat{b}^k$.
- The temporary reserve object price is $\hat{r}^k$.
- If $\hat{b}^k < \hat{r}^k$ then the buyer quits the auction.
- If $\hat{b}^k \geq \hat{r}^k$ then the buyer continues to round $k+1$.

In last round N:

- If $\hat{b}^N < \hat{r}^N$ then no trade.
- If $\hat{r}^N \leq \hat{b}^N < s$ then the buyer pays $p = \frac{(\hat{b}^N + \hat{r}^N)}{2}$.
- If $\hat{b}^k \geq s$ then the buyer pays $p = \frac{(\hat{b}^N + s)}{2}$.

The utility of the seller is:

- If $s > \hat{b}^N$ then utility is 0.
- If $s \leq \hat{b}^N$ then utility $p - S$.

The utility of the buyer is:

- If $\hat{b}^N > s$ then utility is 0.
- If $\hat{b}^N \leq s$ then utility is $B - p$.

The Nash equilibrium exists with
$\hat{b}^N = \hat{r}^N = s \in [B, S]$ otherwise, there will be no equilibrium.

9. Conclusion

In this paper two new autoregression auction formats are proposed namely, simple autoregression auction ($SARA$) and multiple round autoregressive auctions ($MARA$). They both use autoregressive moving average $ARMA(p, q)$ to predict the auction's start price, the recommended object price, the reserve object price and predict the final object price that will be paid by the winner with the highest bid based on bids placed in past auctions and the bids placed by bidders in the current auction. The auctioneer can either adopt $SARA$ or $MARA$ based on the auction objects. $MARA$ is considered an iterative implementation of $SARA$ where the reserve price of any round is the start price of the next round. Therefore, $MARA$ can be converted to $SARA$ in any round of the auction. Also, $MARA$ can easily be more simplified by eliminating some of its steps that makes this auction format easily to implement. The bids of past auctions is an important parameter for both $SARA$ and $MARA$ so a future study is needed to determine how many past auctions is required to predict a good start price, recommended object price, reserve object price and which past auctions to be chosen.

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