Available online at https://asric.africa/natural-sciences
ASRIC Journal on Natural Sciences Vol 3(2) (2023) 147-161

Model for Estimating the Life Expectancy of West African Countries

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Abstract

In this research, the development of a model for estimating life expectancy of the West African countries and the analysis of factors influencing life expectancy of countries was addressed. Thereafter, the proposed model was optimised to confirm whether or not there could be a maximum or minimum life expectancy value. But the result of the model optimisation established that there is neither a maximum nor minimum life expectancy value. Which means that life expectancy of a nation could attain both low and higher value depending on the policies/efforts of the government of that nation. Likewise, the model predictions were contrasted with the online sourced data. And the validation outcome revealed that the absolute difference between the model predictions and the sourced data is very minimal. Equally, the correlation coefficient analysis showed a higher correlation with the real-life data sourced which indicated that the model is performing as expected and thus recommends it as a standard for measuring life expectancy of West African Countries. Moreover, as a recommendation confirmed by the model's variable sensitivity analysis. When targeting areas of life expectancy intervention projects, the income of every nation must be given priority, followed by the quality of health, unemployment rate and economic status of the nation. Lastly, with the proposed model research, it is possible to rank and predict the future life expectancy values of each of the West African countries.

Keywords: Life expectancy, model development, standard of living, Country economic status, validation of the model, correlation coefficient, model optimisation, sensitivity analysis

1.0 Introduction

United Nations (UN) defines life expectancy as an arithmetical measure of the average time an organism is expected to live, based on the year of its birth, its current age and other demographic factors including gender. Thus, whenever the Life expectancy of women in any society is high which means the average age of women in that society will be higher [1]. Likewise, UNICEF defines life expectancy of the number of years new born children would live if subject to the mortality risks prevailing for the cross-sections of population at the time of their birth.

Estimating a country's life expectancy is a worthwhile task to embark on. This is because the standard of living of every individual in a country is tied or resonates on the life expectancy of that country. This remark matches with the remark of [2] that there is a strong correlation and relationship between an individual's standard of living and his/her country's life expectancy.

Similarly, the work of [3] found out that a country's life expectancy is strongly responsible for why the women of that country go into menopause early. They buttressed their remark with the fact that the women in Asian country's average menopause age is above 55years while their African counterpart's average menopause age is less or equal to 45years. They further remarked that in as much as the Asian countries have better and suitable standard of living, the approximate menopause age of their women will continue to be higher and better than those women in African countries (and West Africa to be precise)

All human beings have endlessly sought to better their selves in terms of skills/expertise and to attain a lifestyle which is more and more dignified; as a result, improvement in health has always been the most significant social objectives [4]. Good health is one of the most important assets a human being as a citizen of any nation always has. Healthiness enables us the opportunity to fully widen our capabilities in dreaming for any career prospect on earth. If this very important asset wears away or if it is not well developed, it can form the basis for bodily and emotional weakening, causing impediment in the lives of persons. The previous association could be observed as the relationship between income and health. A set of life cycle models could be proposed to explain how someone's health status could determine future earnings, wealth and expenditure of a person [5].

On a theoretical basis, the works of [6]; [7] and [8] have proposed certain models that incorporate health capital as a considerable variable for economic progress. Based on the findings from their works, and the United Nations (UN) definition that "Life Expectancy is the average number of life years since birth according to the expected rate of mortality by age", then we can remark that, Life expectancy of a nation could be utilised as a broad quantifier of a country's health status [9].

According to a research in 2010, "Life expectancy is defined as the average number of years that a person could expect to live if he or she experienced the age-specific mortality rates prevalent in a given country in a particular year. It does not include the effect of any future decline in age-specific mortality rates. Each country calculates its life expectancy according to somewhat varying methodologies. These methodological differences can affect the exact comparability of reported estimates; as different methods can change a country's measure of life expectancy slightly" [10]. Even with a positive economic development, it will be difficult for governments to maintain or increase the standard of living due to the high population growth rates in almost all the countries in the immediate years to come. There will therefore be a long lasting need for significant foreign assistance to relieve the situation of the people. If peace may come to the countries hit by war there will also be a tremendous need for rehabilitating them using the life expectancy indices of those countries.

2.0 Motivations for the study

A number of situations motivated this study. But the few among them are:

(i) Increasing rate at which women enter menopause early in Nigeria and other West African countries. [11, 12]

(ii) Drastic reduction of individual's standard of living in Nigeria. For example, average standard of living of Nigerians by 2010 was approximately 69.7% but in 2014 Nigeria's average standard of living was as well reduced to 42.4% according to [10] when discussing the Nigerian Bureau of Statistics report of the year 2010. Similarly, [13] in their research had investigated that there is a stronger link between life expectancy and standard of living.

2.1.2 Life Expectancy of West African Countries

According to the 2018 and 2015's World Health Organization (WHO) data, women on the average, live longer than men in all major regions and in all individual countries except for Mali and Swaziland [14]. It is likewise confirmed by UNDF and social affairs in [15]

3.0 Research Methodology

In this chapter we considered the subheadings below as they unfold:

3.1 Model Formulation

In this subsection, some of the basic assumptions needed to make this model conformed to really were made. Also, the study utilised some existing generalisations in literature for the model's variables relationship formulation.

3.1.1 Basic Assumptions

Here, we highlighted the form of mathematical relationships that exist between Life Expectancy and the parameters that influence it in every country considered for this study. Hence, we started addressing each of the parameters as follows:

3.1.1.1 Life Expectancy (Le) versus Nation income (I)

A nation with a high national income tends to have a higher life expectancy and vice versa [16]. Thus we say that life expectancy is directly proportional to income as shown in the equation (3.1) below. And mathematically,

$$Le \;\alpha\; I$$

$$\Rightarrow Le = C_1 I \tag{3.1}$$

3.1.1.2 Life Expectancy (Le) versus Unemployment (U)

In a nation where the unemployment rate is high, the life expectancy of that nation tends to be lower than that of the nations with lower unemployment rate [17]. This means, a drop off in unemployment ratio will lead to an equivalent increase in life expectancy rate. Thus there is an inverse proportional relationship between life expectancy and unemployment rate as shown in equation (3.2).

$$Le \;\alpha \frac{1}{U}$$

$$\Rightarrow Le = \frac{C_2}{U} \tag{3.2}$$

3.1.1.3 Life Expectancy (Le) versus Quality of health (Qh) of a Country

Another good factor that determines life expectancy of a country is the quality of health care of the nation. If a nation has good health care opportunities, that nation tends to have better life expectancy [12]. This means that an increase in the Quality of health (Qh) rate leads to a corresponding increase in life expectancy ratio. Thus there is a direct proportional relationship between life expectancy and Quality of health (Qh) rate as shown in equation (3.3).

$$Le \,\alpha\, Qh$$

$$Le = C_3 Qh \tag{3.3}$$

3.1.1.4 Life expectancy (Le) versus Economic Status (E) of a Country

Similarly, the nations with better and higher Economic status (E) have a better life expectancy. This could be interpreted as a direct proportionality relationship between Le and E given that an increasing Economic status (E) of a nation, translates to a corresponding increase in the life expectancy ratio of that country [18]. Thus their mathematical relationship is as shown in equation (3.4).

$$Le \,\alpha\, E$$

$$Le = C_4 E + C_5 \tag{3.4}$$

3.1.2 Establishment of the Model Parameter Relationships

From the postulations in the model assumptions section, we could add up equations (3.1), (3.2), (3.3) and (3.4) in order to obtain:

$$4Le = C_1 I + \frac{C_2}{U} + C_3 Qh + C_4 E + C_5$$

$$Le = \frac{C_1}{4} I + \frac{C_2}{4U} + \frac{C_3}{4} Qh + \frac{C_4}{4} E + \frac{C_5}{4}$$

Let: $\frac{C_1}{4} = \alpha, \frac{C_2}{4} = \gamma, \frac{C_3}{4} = \phi, \frac{C_4}{4} = a, and\ C_5 = b$

Thus,

$$Le = \alpha I + \frac{\gamma}{U} + \phi Qh + aE + b \tag{3.5}$$

Where; Le = Life Expectancy, I = National Income of a country, U = Unemployment rate of a country, Qh = Quality of health,

E = Nation's Economy Status/ranking.
And $\alpha, \gamma, \phi, a \; and \, b$ are model's constants

3.2 Computation of the Model Equation Constants

We apply least Squares method on equation (3.5) so as to obtain the values for our model equation constants.

$$Z_{\min} = \min \sum_{i=1}^{N} \left(Le_i - \alpha I_i - \frac{\gamma}{U_i} - \phi Qh_i - aE_i - b \right)^2 \quad (3.6)$$

Differentiating equation (3.6) partially with respect to each of the distinct model constants yields;

$$\frac{\partial Z}{\partial \alpha} = -2\sum_{i=1}^{n} \left(Le_i - \alpha I_i - \frac{\gamma}{U_i} - \phi Qh_i - aE_i - b \right) I_i \quad (3.7)$$

$$\frac{\partial Z}{\partial \gamma} = -2\sum_{i=1}^{n} \left(Le_i - \alpha I_i - \frac{\gamma}{U_i} - \phi Qh_i - aE_i - b \right) \frac{1}{U_i} \quad (3.8)$$

$$\frac{\partial Z}{\partial \phi} = -2\sum_{i=1}^{n} \left(Le_i - \alpha I_i - \frac{\gamma}{U_i} - \phi Qh_i - aE_i - b \right) Qh_i \quad (3.9)$$

$$\frac{\partial Z}{\partial a} = -2\sum_{i=1}^{n} \left(Le_i - \alpha I_i - \frac{\gamma}{U_i} - \phi Qh_i - aE_i - b \right) E_i \quad (3.10)$$

$$\frac{\partial Z}{\partial b} = -2\sum_{i=1}^{n} \left(Le_i - \alpha I_i - \frac{\gamma}{U_i} - \phi Qh_i - aE_i - b \right) \quad (3.11)$$

But at optimal point, $\frac{\partial Z}{\partial b} = \frac{\partial Z}{\partial a} = \frac{\partial Z}{\partial \phi} = \frac{\partial Z}{\partial \gamma} = \frac{\partial Z}{\partial \alpha} = 0$

Thus equation (3.7) gives,

$$-2\sum_{i=1}^{n} \left(Le_i - \alpha I_i - \frac{\gamma}{U_i} - \phi Qh_i - aE_i - b \right) I_i = 0$$

$$\Rightarrow \sum_{i=1}^{n} Le_i I_i - \alpha \sum_{i=1}^{n} I^2{}_i - \gamma \sum_{i=1}^{n} \frac{I_i}{U_i} - \phi \sum_{i=1}^{n} Qh_i I_i - a \sum_{i=1}^{n} E_i I_i - b \sum_{i=1}^{n} I_i = 0 \quad (3.12)$$

Similarly, (3.8) gives;

$$\sum_{i=1}^{n} \frac{Le_i}{U_i} - \alpha \sum_{i=1}^{n} \frac{I_i}{U_i} - \gamma \sum_{i=1}^{n} \frac{1}{U^2{}_i} - \phi \sum_{i=1}^{n} \frac{Qh_i}{U_i} - a \sum_{i=1}^{n} \frac{E_i}{U_i} - b \sum_{i=1}^{n} \frac{1}{U_i} = 0 \quad (3.13)$$

Also, equation (3.9) gives;

$$\Rightarrow \sum_{i=1}^{n} Le_i Qh_i - \alpha \sum_{i=1}^{n} I_i Qh_i - \gamma \sum_{i=1}^{n} \frac{Qh_i}{U_i} - \phi \sum_{i=1}^{n} Qh^2{}_i - a \sum_{i=1}^{n} E_i Qh_i - b \sum_{i=1}^{n} Qh_i = 0 \quad (3.14)$$

Then equation (3.10) yields;

$$\sum_{i=1}^{n} Le_i E_i - \alpha \sum_{i=1}^{n} I_i E_i - \gamma \sum_{i=1}^{n} \frac{E_i}{U_i} - \phi \sum_{i=1}^{n} Qh_i E_i - a \sum_{i=1}^{n} E^2{}_i - b \sum_{i=1}^{n} E_i = 0 \quad (3.15)$$

Finally, equation (3.10) yields;

$$\sum_{i=1}^{n} Le_i - \alpha \sum_{i=1}^{n} I_i - \gamma \sum_{i=1}^{n} \frac{1}{U_i} - \phi \sum_{i=1}^{n} Qh_i - a \sum_{i=1}^{n} E_i - b \sum_{i=1}^{n} \quad = 0 \quad (3.16)$$

Where I = 1, 2, 3 …n. But since this study is concerned with only West African countries, n =17. Meanwhile equation (3.12), (3.13), (3.14), (3.15), and (3.16) are to be solved by Gauss' Elimination method to obtain $\alpha, \gamma, \phi, a \ and \, b$.

3.3 Research Data Sources

For this study, the research data used was the secondary sourced data. This is because the data used were obtained from already published journals and organization websites such as WHO, UNICEF, United Nation and other African Nation's data websites. Furthermore, the data for this study was restricted to only the 17 West Africa countries and for the year, 2018 to be specific.

Table 1: The West Africa Countries and their Information Obtained

Countries	National income(I) US$ dollars	Unemployment rate(U) measure in %	Health ranking
Nigeria	2080	18.8	187
Ghana	1490	2.4	135
Mali	770	7.9	163
Burkinafaso	610	6.3	132
Ivory coast	1540	2.6	137
Gambia	450	9.5	139
Liberia	380	2.4	186
Niger	360	0.4	170
Mauritania	1100	11.8	162
Benin	800	2.5	97
Cape verde	2990	9	113
Guinea	820	4.5	161
Guinea bissau	660	6.1	176
Senegal	950	15.7	59
Togo	610	1.8	152
Seirra leone	510	4.5	191
Sao tome & p	1770	13.5	133
	$\sum_{i=1}^{n} I_i = 17890$	$\sum_{i=1}^{n} U_i = 119.7$	Sum=2493

Source: online sources from the following websites:

- https://www.afdb.org/fileadmin/uploads/afdb/Documents/Publications/2019AEO/REO_2019_-_West_africa.pdf
- https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?end=2018&locations=ZG&most_recent_year_desc=true&s
- https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=&cad=rja&uact=8&ved=2ahUKEwir2f-y3KCAAxUEXUEAHSuXDr4QFnoECCQQAQ&url=https%3A%2F%2Fwww.worldbank.org%2Fen%2Fregion%2Fafr%2Foverview&usg=AOvVaw3984euV-sA3URoTMKVtmYf&opi=89978449
- https://www.afdb.org/fileadmin/uploads/afdb/Documents/Publications/2018AEO/African_Economic_Outlook_2018_West-

Table 2: The West Africa Countries and their Information Obtained (continuation)

Quality of health (Qh)	Economical Ranking	Economic Status	Life Expectancy
0.026041667	31	0.838541667	55.5
0.296875	85	0.557291667	63
0.151041667	118	0.385416667	58
0.3125	124	0.354166667	60.5
0.286458333	87	0.546875	55
0.276041667	178	0.072916667	62
0.03125	162	0.15625	63
0.114583333	142	0.260416667	60
0.15625	154	0.197916667	64
0.494791667	138	0.28125	61
0.411458333	169	0.119791667	73
0.161458333	139	0.276041667	59.5
0.083333333	174	0.09375	59.5
0.692708333	114	0.40625	67
0.208333333	151	0.213541667	61
0.005208333	159	0.171875	53
0.307291667	185	0.036458333	69
$\sum_{i=1}^{n} Qh_i = 4.015625$	2310	$\sum_{i=1}^{n} E_i = 4.96875$	$\sum_{i=1}^{n} Le_i = 1044$

Source: online sources from the following websites

- https://www.afdb.org/fileadmin/uploads/afdb/Documents/Publications/2019AEO/REO_2019_-_West_africa.pdf
- https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?end=2018&locations=ZG&most_recent_year_desc=true&s
- https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=&cad=rja&uact=8&ved=2ahUKEwir2f-y3KCAAxUEXUEAHSuXDr4QFnoECCQQAQ&url=https%3A%2F%2Fwww.worldbank.org%2Fen%2Fregion%2Fafr%2Foverview&usg=AOvVaw3984euV-sA3URoTMKVtmYf&opi=89978449
- https://www.afdb.org/fileadmin/uploads/afdb/Documents/Publications/2018AEO/African_Economic_Outlook_2018_West-

Table 3: Computation of the Equation constants Using Secondary Data sourced in the Table 1 & 2

Le*I	I^2	I/U	I*Qh	E*I	Le/U	1/U^2	Qh/U
115440	4326400	110.638	54.16667	1744.167	2.95213	0.002829	0.001385
93870	2220100	620.833	442.3438	830.3646	26.25	0.173611	0.123698
44660	592900	97.4684	116.3021	296.7708	7.34177	0.016023	0.019119
36905	372100	96.8254	190.625	216.0417	9.60318	0.025195	0.049603
84700	2371600	592.308	441.1458	842.1875	21.1539	0.147929	0.110176
27900	202500	47.3684	124.2188	32.8125	6.52632	0.01108	0.029057
23940	144400	158.333	11.875	59.375	26.25	0.173611	0.013021
21600	129600	900	41.25	93.75	150	6.25	0.286458
70400	1210000	93.2203	171.875	217.7083	5.42373	0.007182	0.013242

48800	640000	320	395.8333	225	24.4	0.16	0.197917
218270	8940100	332.222	1230.26	358.1771	8.11111	0.012346	0.045718
48790	672400	182.222	132.3958	226.3542	13.2222	0.049383	0.03588
39270	435600	108.197	55	61.875	9.75410	0.026874	0.013661
63650	902500	60.5096	658.0729	385.9375	4.26752	0.004057	0.044122
37210	372100	338.889	127.0833	130.2604	33.8889	0.308642	0.115741
27030	260100	113.333	2.65625	87.65625	11.7778	0.049383	0.001157
122130	3132900	131.111	543.9063	64.53125	5.11111	0.005487	0.022762
$\sum_{i=1}^{n} Le_i * I_i = 1124565$	$\sum_{i=1}^{n} I^2{}_i =$ 26925300	$\sum_{i=1}^{n} \frac{I_i}{U_i} =$ 4303.479	$\sum_{i=1}^{n} Qh_i * I_i =$ 4739.01	$\sum_{i=1}^{n} E_i * I_i =$ 5872.969	$\sum_{i=1}^{n} \frac{Le_i}{U_i} =$ 366.0337	$\sum_{i=1}^{n} \frac{1}{U^2{}_i} =$ 7.423633	$\sum_{i=1}^{n} \frac{Qh_i}{U_i} =$ 1.122717

Table 4: Computation of the Equation constants Using the Secondary Data in Table 1 & 2 (Continued)

E/U	1/U	Le*Qh	Qh^2	E*Qh	Le*E	E^2
0.044603	0.053191	1.445313	0.000678	0.021837	46.53906	0.703152
0.232205	0.416667	18.70313	0.088135	0.165446	35.10938	0.310574
0.048787	0.126582	8.760417	0.022814	0.058214	22.35417	0.148546
0.056217	0.15873	18.90625	0.097656	0.110677	21.42708	0.125434
0.210337	0.384615	15.75521	0.082058	0.156657	30.07813	0.299072
0.007675	0.105263	17.11458	0.076199	0.020128	4.520833	0.005317
0.065104	0.416667	1.96875	0.000977	0.004883	9.84375	0.024414
0.651042	2.5	6.875	0.013129	0.029839	15.625	0.067817
0.016773	0.084746	10	0.024414	0.030924	12.66667	0.039171
0.1125	0.4	30.18229	0.244819	0.13916	17.15625	0.079102
0.01331	0.111111	30.03646	0.169298	0.049289	8.744792	0.01435
0.061343	0.222222	9.606771	0.026069	0.044569	16.42448	0.076199
0.015369	0.163934	4.958333	0.006944	0.007813	5.578125	0.008789
0.025876	0.063694	46.41146	0.479845	0.281413	27.21875	0.165039
0.118634	0.555556	12.70833	0.043403	0.044488	13.02604	0.0456
0.038194	0.222222	0.276042	2.71E-05	0.000895	9.109375	0.029541
0.002701	0.074074	21.20313	0.094428	0.011203	2.515625	0.001329
$\sum_{i=1}^{n} \frac{E_i}{U_i} =$ 1.720669	$\sum_{i=1}^{n} \frac{1}{U_i} =$ 6.059275	$\sum_{i=1}^{n} Le_i * Qh_i =$ 254.9115	$\sum_{i=1}^{n} Qh^2{}_i =$ 1.470893	$\sum_{i=1}^{n} E_i * Qh_i =$ 1.177436	$\sum_{i=1}^{n} Le_i * E_i =$ 297.9375	$\sum_{i=1}^{n} E^2{}_i =$ 2.143446

Hence, using the summation values from the last rows of tables1, 2, 3 and 4 we have that;

$\sum_{i=1}^{n} I_i = 17890$, $\sum_{i=1}^{n} U_i = 119.7$, $\sum_{i=1}^{n} Qh_i = 4.015625$, $\sum_{i=1}^{n} E_i = 4.96875$, $\sum_{i=1}^{n} Le_i = 1044$, $\sum_{i=1}^{n} Le_i * I_i =$ 1124565, $\sum_{i=1}^{n} I^2{}_i = 26925300$, $\sum_{i=1}^{n} \frac{I_i}{U_i} = 4303.479$, $\sum_{i=1}^{n} Qh_i * I_i = 4739.01$,

$\sum_{i=1}^{n} E_i * I_i = 5872.969$, $\sum_{i=1}^{n} \frac{Le_i}{U_i} = 366.0337$, $\sum_{i=1}^{n} \frac{1}{U^2{}_i} = 7.423633$, $\sum_{i=1}^{n} \frac{Qh_i}{U_i} = 1.122717$,

$\sum_{i=1}^{n} \frac{E_i}{U_i} = 1.720669$, $\sum_{i=1}^{n} \frac{1}{U_i} = 6.059275$, $\sum_{i=1}^{n} Le_i * Qh_i = 254.9115$, $\sum_{i=1}^{n} Qh^2{}_i = 1.470893$,

$\sum_{i=1}^{n} E_i * Qh_i = 1.177436$, $\sum_{i=1}^{n} Le_i * E_i = 297.9375$, and $\sum_{i=1}^{n} E^2{}_i = 2.143446$

Therefore, substituting the above values into equation (3.12), (3.13), (3.14), (3.15), and (3.16), we have;

$$1124565 - 26925300\alpha - 4303.479\gamma - 4739.01\phi - 5872.969a - 17890b = 0 \quad (3.17)$$

$$366.0337 - 4303.479\alpha - 7.4236\gamma - 1.1227\phi - 1.7207a - 6.0593b = 0 \quad (3.18)$$

$$254.9115 - 4739.01\alpha - 1.1227\gamma - 1.4709\phi - 1.1774a - 4.0156b = 0 \quad (3.19)$$

$$297.9375 - 5872.969\alpha - 1.7207\gamma - 1.1774\phi - 2.1435a - 4.9688b = 0 \quad (3.20)$$

$$1044 - 17890\alpha - 6.0593\gamma - 4.0156\phi - 4.9688a - 17b = 0 \quad (3.21)$$

Hence, solving equation (3.17), (3.18), (3.19), (3.20), and (3.21) above we obtain;

$\alpha = 0.003726967359$

$\gamma = 0.9358468269$

$\phi = 12.89160686$

$a = -13.89789235$

$b = 58.17307578$

Also, putting the values of α, γ, ϕ, a *and* b into equation (3.5) yields,

$$Le = 0.003726967359 * I + \frac{0.9358468269}{U} + 12.89160686 * Qh - 13.89789235 * E + 58.17307578 \quad (3.22)$$

Thus equation (3.22) is the developed model for estimating a country's life expectancy.

4.0 Discussion of Results

In this section, the model results were validated and contrasted with the real-life data sourced from the internet using the correlation coefficient approach. Similarly, the graphical profiles reflecting the trends curve between the model prediction result against the real-life data were presented for this study.

4.1 Validation of the Model

After developing the model, we test its prediction by comparing its results with the data obtain to see if our model conforms to reality. The test further confirmed that the model could be adopted for its purpose of formulation since the absolute difference obtained between the model's data generated and the actual sourced data is approximately less than 5 units (as shown in table 4 below). However, the little difference perhaps, may be as a result of the few more factors that affect life expectancy which are not captured in this study.

Table 5: The Tabular Validation of the Model

This table shows the difference between the sourced Life expectancy data and our model predictions

Life Expectancy (Le) [from sourced data]	**Life Expectancy (Le)** [our model Predictions]	**Absolute error**
55.5	54.6567	0.843296
63	60.19821	2.80179
58	57.75199	0.248007
60.5	59.70153	0.79847
55	60.36504	5.365045
62	62.49395	0.493954
63	58.21058	4.789423
60	59.71232	0.287678
64	61.61574	2.384262
61	63.99887	2.998866
73	73.0602	0.060199
59.5	59.68221	0.182215
59.5	60.55766	1.057665
67	65.05741	1.942592
61	60.68441	0.315587
53	57.96024	4.960239
69	68.29392	0.706081

4.2 Correlation coefficient between Gathered Data and Model Estimated Life expectancy

By formula definition;

$$\text{Correlation coefficient (r)} = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - \left(\sum x\right)^2][n\sum y^2 - \left(\sum y\right)^2]}} \qquad (3.23)$$

Hence, computing the values of the summation values from the formula we have;

Table 6: Correlation Coefficient computation table

LIFE EXPECTANCY (Le)	Model estimated Life Expectancy (MLe)	Le^2	MLe^2	Le*MLe
55.5	54.6567	3080.25	2987.355	3033.447
63	60.19821	3969	3623.824	3792.487
58	57.75199	3364	3335.292	3349.615
60.5	59.70153	3660.25	3564.273	3611.943
55	60.36504	3025	3643.938	3320.077
62	62.49395	3844	3905.494	3874.625
63	58.21058	3969	3388.472	3667.267
60	59.71232	3600	3565.561	3582.739
64	61.61574	4096	3796.499	3943.407
61	63.99887	-3721	4095.855	3903.931
73	73.0602	5329	5337.793	5333.395
59.5	59.68221	3540.25	3561.966	3551.091
59.5	60.55766	3540.25	3667.23	3603.181
67	65.05741	4489	4232.467	4358.846
61	60.68441	3721	3682.598	3701.749
53	57.96024	2809	3359.389	3071.893
69	68.29392	4761	4664.06	4712.28

$\sum_{i=1}^{17} Le_i = 1044$	$\sum_{i=1}^{17} MLe_i =$ 1044.001	$\sum_{i=1}^{17} Le^2{}_i = 64518$	$\sum_{i=1}^{17} MLe^2{}_i =$ 64412.07	$\sum_{i=1}^{17} Le_i * MLe_i =$ 64411.97

Thus;

$$r = \frac{17*64411.97 - 1044*1044.001}{\sqrt{[17*64518 - 1044^2][17*64412.07 - 1044.001^2]}}$$

$$r = \frac{5066.446}{5889.133}$$

$$r = 0.860 \quad \Rightarrow r = 86\%$$

Remark: from the correlation coefficient validation analysis of the model carried out using the above table 4 data, a correlation coefficient of 0.860 was obtained. Based on this coefficient value, we could therefore say that our model performs up to 86% correlation with real life data used. And thus, we conclude that there is a high degree of correlation, which could recommend the model as a standard model for predicting life expectancy.

4.3 Graphical Correlation between the Models life expectancy and gathered data.

Under this subsection, we utilised graphical analysis to buttress the level of correlation that exist between the Model predictions and real life data sourced for the study.

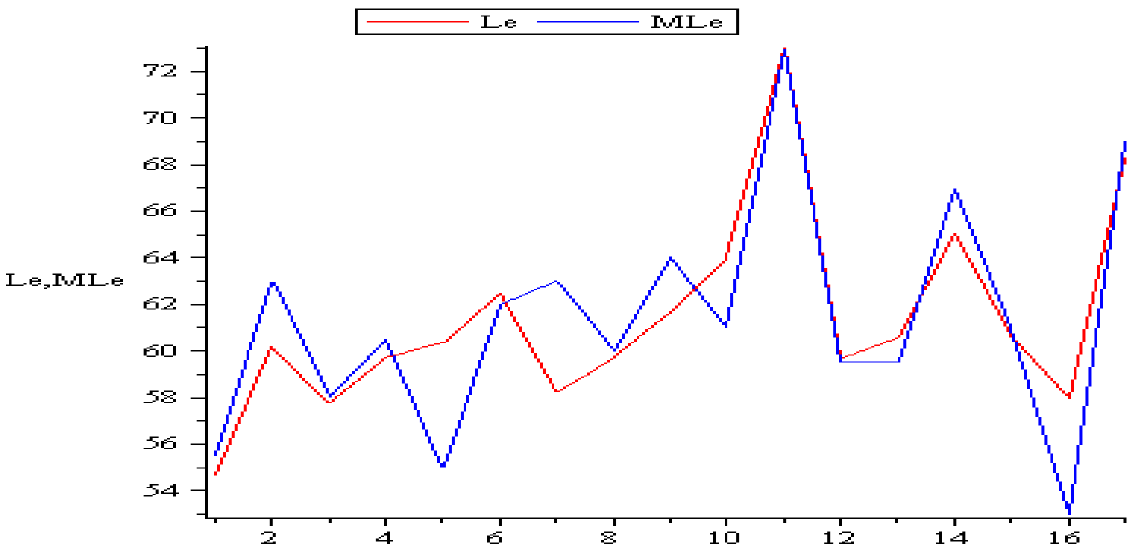

Fig 1: Graph of the model's Life Expectancy value versus the sourced data for 2018.

Remarks: from the figure1 above, the model data and the sourced real-life data followed similar trend and pattern.

4.4 Optimization of the Model

The optimization here is the mathematical method of determining the best possible option (the best possible way of maximizing or minimising the West African's individual nation's life expectancy), subject to some particular set of constraints.

Thus, to achieve this, we recall from our model equation (3.5) above and let,

$$Le = \alpha I + \frac{\gamma}{U} + \phi Qh + aE + b \tag{4.1}$$

where from equation (3.1)

$$Le = C_1 I \Rightarrow I = \frac{Le}{4\alpha} \quad (4.2)$$

Since $\alpha = \frac{C_1}{4}$

Also from equation (3.2)

$$Le = \frac{C_2}{U} \Rightarrow U = \frac{4\gamma}{Le} \quad (4.3)$$

Since $C_2 = 4\gamma$,

Hence putting (4.3) and (4.2) into (4.1) gives,

$$Le = \frac{\alpha Le}{4\alpha} + \frac{\gamma Le}{4\gamma} + \phi Qh + aE + b$$

$$\Rightarrow Le\left(1 - \frac{1}{4} - \frac{1}{4}\right) = \phi Qh + aE + b$$

$$\Rightarrow \frac{Le}{2} = \phi Qh + aE + b$$

$$thus, \; Le = 2\phi Qh + 2aE + 2b \quad (4.4)$$

Therefore, equation (4.4) is our reduced model equation in 2-variables (i.e., Quality of health and Economic Status).

4.4.1 Observation of the Nature of the Model Extreme Values

This stage is helpful in knowing whether the model's extreme value is maximum or a minimum. Generally, given a continuous function f(x,y) that obeys the continuity of partial derivatives as we let

$$A = \frac{\partial^2 f}{\partial x^2}; \quad B = \frac{\partial^2 f}{\partial x \partial y}; \; C = \frac{\partial^2 f}{\partial y^2};$$

Then if:

1. $B^2 - AC < 0,$ Then f(x,y) has extreme value (x_0, y_0) and minimum if A>0 and it is maximum if A< 0.
2. $B^2 - AC > 0,$ Or $AB < C^2$ then f(x,y) has no extreme value. And it implies that at the point (x_0, y_0) it is saddle.
3. $B^2 - AC = 0$, then one has no knowledge about its extreme values.

Thus, applying the above rule to our reduced model equation variable, we have:

$$\frac{\partial Le}{\partial Qh} = 2\phi$$

$$\Rightarrow A = \frac{\partial^2 Le}{\partial Qh^2} = 0 \qquad (4.6)$$

Also,

$$\frac{\partial Le}{\partial E} = 2a$$

$$\Rightarrow C = \frac{\partial^2 Le}{\partial E^2} = 0 \qquad (4.7)$$

similarly,

$$B = \frac{\partial^2 Le}{\partial Qh \partial E} = \frac{\partial}{\partial Qh}\left(\frac{\partial Le}{\partial E}\right) = \frac{\partial}{\partial Qh}(2a) = 0 \qquad (4.8)$$

Then,

$$B^2 - AC = 0 \qquad (4.9)$$

Thus from the result in equation (4.9) above, we could say that there is neither a minimum nor maximum number of age that an individual in a particular West African country is expected to live and get to (as a life expectancy value). And this result implies that some West African nations that have very low life expectancy value now can still have their life expectancy value moved higher (if the nation's economic status, quality of health and other health factors are improved). Similarly, those nations that have very high life expectancy value now, could still have their life expectancy value dropped downwards (if the nation's economic status, quality of health and other health factors are not constantly improved upon).

4.5 Sensitivity Analysis of the Model and the socioeconomic implication

In order to be able to highlight some of the socioeconomic implication of the study, the model's variable sensitivity analysis was carried out.

From the Variable Sensitivity Analysis using the Maple 18 computation result (as attached/shown in the Appendix section of this study) the following observations as reported in the table 7 below were made:

Table 7: Variable Sensitivity Analysis of the Proposed Menopause Model

S/N	Variables	Sensitivity Index	Ranks
1	National Income (I)	0.06414478311	1st
2	Quality of Health (Qh)	0.04980021924	2nd
3	Unemployment (U)	- 0.001086856995	3rd
4	Economy Status (E)	- 0.06643884375	4th

From the table however;

- It was observed that the model's variable with the highest impact on the Life expectancy of each West African nation is the National Income (I) of each nation.
- It was also followed by the Quality of health (Qh) of each of the West African nations.
- Similarly, the 3rd in the hierarchy of impact on the model is the citizen's Unemployment rate (U).
- And the last on the list is the Economy Status (E) of the citizens.

Remarks from subsection 4.5:

Nevertheless, from the results as shown in the table 6, the National Income (I) of each of the West African nations has the highest impact on the Life expectancy of each nation than the quality of health (which is 2nd on the list) because, every of the West African nations needs income to be able to provide improved quality of health to her citizens. Similarly, the next item in terms of impact on the model is the quality of health (Qh) of citizens. But Qh has the highest impact than the citizens Unemployment rate (U) because, quality of health is needed for the citizen's to be gainfully-employed-and-remain in the employment.

So also, unemployment rate is higher in hierarchy than the nation's Economy Status because, when people are gainfully employed, it directly tells on how long they could live (which is the nation's life expectancy in brief). This is also because, with their gainful employment, they could meet their daily basic needs; which consequently tends to prolong the lives of each citizens.

4.6 Discussion of the result in Details (Discussion of the model results in linear form)

- The Life Expectancy values from the model Prediction in table4 were obtained by substituting the values of table1 into our formulated model in equation (3.22). While the absolute error is obtained by subtracting the model's predicted results from the online real-life data on Life Expectancy of the West African Nations obtained.
- The correlation coefficient values in the table 6 was obtained with the aid of equation (3.23) by replacing y for Model estimated Life Expectancy (MLe) and x for life expectancy (Le) of the sourced data.
- The constants $\alpha, \gamma, \phi, a \ \ and\, b$ of equation (3.5) were obtained via the Least Squares Method in order to arrive at the formulated model in equation (3.22)
- Since every of the persons in a country cannot be 100% employed, then the model will be usable in the domain/region where the variable, unemployment rate (U) is not equal to zero. And thus, whenever some of the model assumptions are violated, then the region/range of definition of the model would be affected which could adversely affect the model’s predictions.
- The model’s optimisation approach followed the simple second derivative process of either minimising or maximising a function.
- The socioeconomic implication of the study is well highlighted in the subsection 4.5 above. It is summarily connected to how citizens of the individual West African nations that could bring development to the West African activities have their life span shortened due to an unbalanced quality of life and health services in those countries.

4.7 Contribution of the Study

(i) Provision of quantitative tool for prediction

(ii) Proposing a ranking tool for West African states based on their citizen's life expectancy and quality of life.

(iii) via optimisation analysis, the study has ascertained that, there is neither a constant minimum nor maximum number of age that an individual in a particular West African nation is expected to live until (as a life expectancy value). Implying that some West African nations that have very low life expectancy value now could still have theirs moved upwards (if the nation’s economic status, Quality of health and other factors are improved). Similarly, those nations that have very high life expectancy value could still have theirs dropped downwards (if the nation’s economic status, Quality of health and other factors are not constantly improved upon by their respective government).

(iv) Provision of a research quantitative tool that could be replicated in other part of Africa and the entire world at large (following the simple steps highlighted by the study).

(v) Provision of a variable ranking tool for the model in terms of how impactful or the degree of impact each of the variable has on the model. This is shown in section 4.5 of the study.

5.0 Conclusion

In this study, after formulating the model, we validated the accuracy of the model by comparing it with real-life data sourced for this study and found out that the model has a higher correlation with reality data utilised. From the various levels of the model analysis, this study has established that long life of citizens is a vital goal that every government of countries should desire to pursue and achieve. And in order to live longer, individuals and government of countries has to take cognizance of those factors that affect human life expectancy as highlighted in this study. Moreover, as confirmed in the model's variable sensitivity analysis, when targeting areas of life expectancy intervention projects, the income of every nation must be given priority, followed by the quality of health, unemployment rate and economic status of the nation.

Similarly, with the formulated model any West African country representative could input its nation's data into the model and obtain an estimated life expectancy age /value for her citizens. It could as well, help West African nations to checkmate and predict whether the future years' life expectancy is favourable or not.

Lastly, the optimization result of the model in equation (4.9) showed that there is neither any specific maximum nor minimum life expectancy age value that a citizen of a country could live up till or outlive. It means that any country's life expectancy has the capacity to go higher (when a country decides to improve the standard of living, economic status, quality of health and other welfare factors of her citizens constantly) and could drop low (when a country fails to advance the course of standard of living along with the entire welfare factors of her citizens.

5.1 Recommendations

The following recommendations were made based on the results/findings obtained from the study:

(i) Using the import from the sensitivity analysis of the model, it was observed that for every West African States to be able to maintain a very favourable Life Expectancy, it must constantly improve on her National Income (by ensuring a fair GDP boost and balance of trade).
(ii) It must likewise create an enabling environment for improving quality of health of her citizens.
(iii) The above must also be followed by the provision of employment opportunities for the citizens.
(iv) It must finally maintain a high economic status among other nations of the world.
(v) Lastly, in view of all the basic modelling questions answered by this study, this proposed model could be utilised for ranking West African nations in terms of life expectancy.

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